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Section :: "A"

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Subject :: LINEAR ALGEBRA

Q 1.

i) $A = \begin{bmatrix} 1 & 2 & \text{2nd - ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$ID = 16027$

2nd number $= 2$

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$A^{adj} = ?$

Solution. Replace all values its cofactors then take transpose also equal to adjoint of A.

$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = (1) \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6 - 1 = 5$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = (-1)(4-3) = -1$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = (1)(2-9) = -7.$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} = (-1)(4-2) = -2.$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = (1)(2-6) = -4.$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (-1)(1-6) = 5$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = (1)(2-6) = -4$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = (-1)(1-4) = 3.$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (1)(3-4) = -1$$

$$A_{cofactor} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -2 & -4 & 5 \\ -4 & 3 & -1 \end{bmatrix}$$

Now transpose

$$A_{adj} = A^{trans} = \begin{bmatrix} 5 & -2 & -4 \\ -1 & -4 & 3 \\ -7 & 5 & -1 \end{bmatrix}$$

Ans

Q1:

ii) $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

solution. $A_{adj} = ?$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = (1)(-8+16) = 8$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = (-1)(16-40) = 24$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (1)(-4+5) = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = (-1)(32+10) = -42.$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = (1)(24-25) = -1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = (-1)(-6-20) = 26.$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = (1)(32+5) = 37$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = (-1)(24-10) = -14.$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = (1)(-3-8) = -11$$

Now replace all values its cofactors.

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Now transpose.

$$B_{adj} = B^t = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

Ans

Q2: Find the cofactors of A_{21}, A_{31}, A_{33} if.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Solution

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-1)(-4+9) = -5$$

$$\boxed{A_{21} = -5}$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (1)(-2-9) = -11$$

$$\boxed{A_{31} = -11}$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = (1)(3-4) = -1$$

$$\boxed{A_{33} = -1}$$

—— x —— x —— x ——

Q3

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \text{ \& } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

solution:

step 1: $\boxed{|A - \lambda I| = 0} \to$ formula.

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

NOW taken Determinent.

$$2-\lambda\,((3-\lambda)(2-\lambda) - 2) - 1\,(2 - (2-\lambda)) + 1\,(1 + 1(3-\lambda)) = 0$$

$$2-\lambda\,(6 - 3\lambda - 2\lambda + \lambda^2 - 2) - 1\,(2 - 2 + \lambda) + 1\,(1 + 3 - \lambda) = 0$$

$$2-\lambda\,(\lambda^2 - 5\lambda + 4) - 1\,(\lambda) + 1\,(4 - \lambda) = 0$$

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$2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda - \lambda + 4 - \lambda = 0$

by ordering

$-\lambda^3 + 7\lambda^2 - 16\lambda + 12 = 0$

Xing by (−).

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0 \longrightarrow$ (A)

Now put $\lambda - 2 = 0$

then $\lambda = 2$ in above eq (A).

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

put $\lambda = 2$.

$(2)^3 - 7(2)^2 + 16(2) - 12 = 0$

$8 - 7(4) + 32 - 12 = 0$

$8 - 28 + 32 - 12 = 0$

$-20 + 20 = 0$

$0 = 0$.

So Eigen value: $\lambda = 2$ or $\lambda - 2 = 0$

Ans