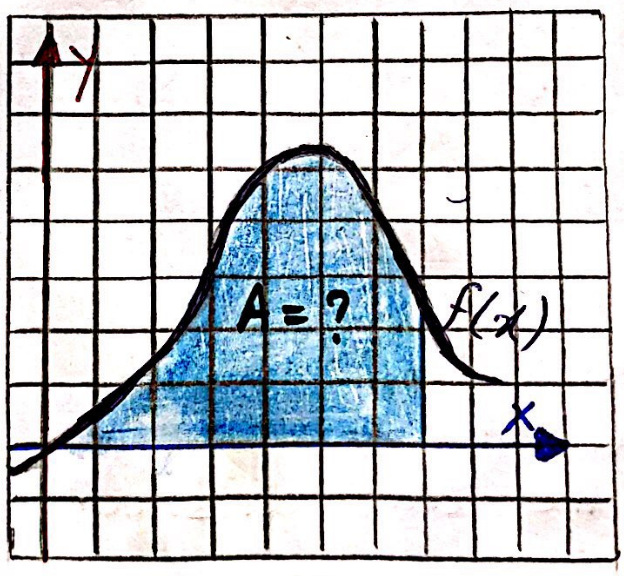
NAME	HARIS IQBAL
ID	7926
SECTION	"A"
SUBJECT	NUMERICAL ANALYSIS
ASSIGNMENT	1

Introduction To Integration:-

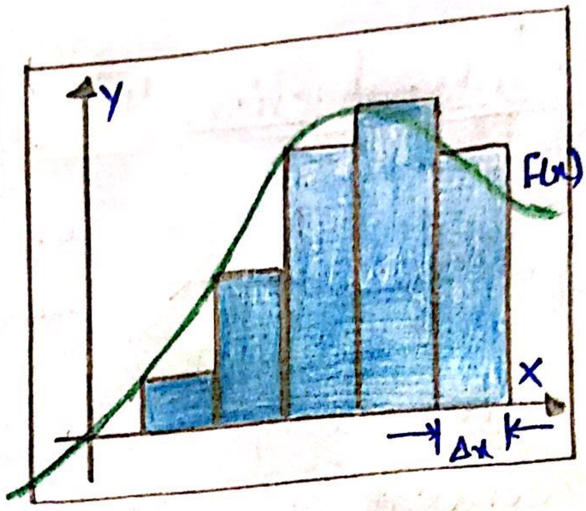
Integration is a way of adding slices to find the whole.

Integration can be used to find areas, volumes, central points and many useful things. But it is easiest to start with finding the area under the curve of a function lik this:

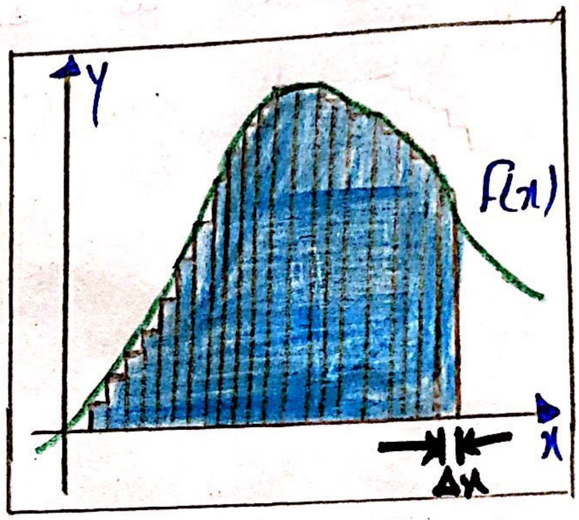


What is the area under $y = f(x)$?

⇒ We could calculate the function at a few points and add up slices of width Δx like this (but the answer won't be very accurate):

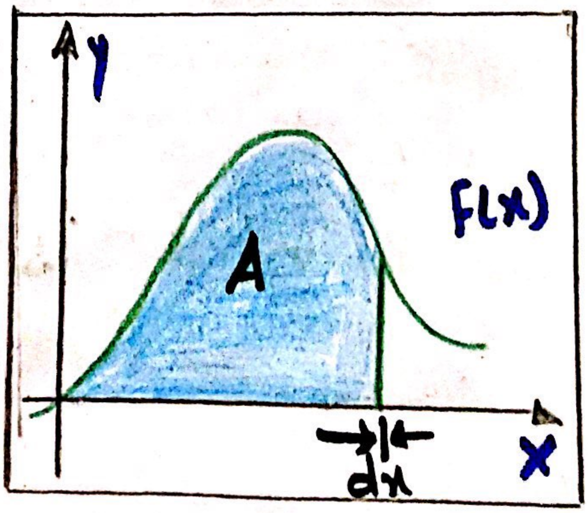


⇒ We can make Δx a lot smaller and add up many small slices (answer is getting better):



⇒ And as the slices approach zero in width, the answer approaches the true answer.

⇒ We now write dx to mean the Δx slices are approaching zero.



Trapezoidal Rule:-

We know from a previous lesson that we can use Riemann Sums to evaluate a definite integral $\int_a^b f(x)\,dx$.

Riemann Sums use rectangles to approximate the area under a curve.

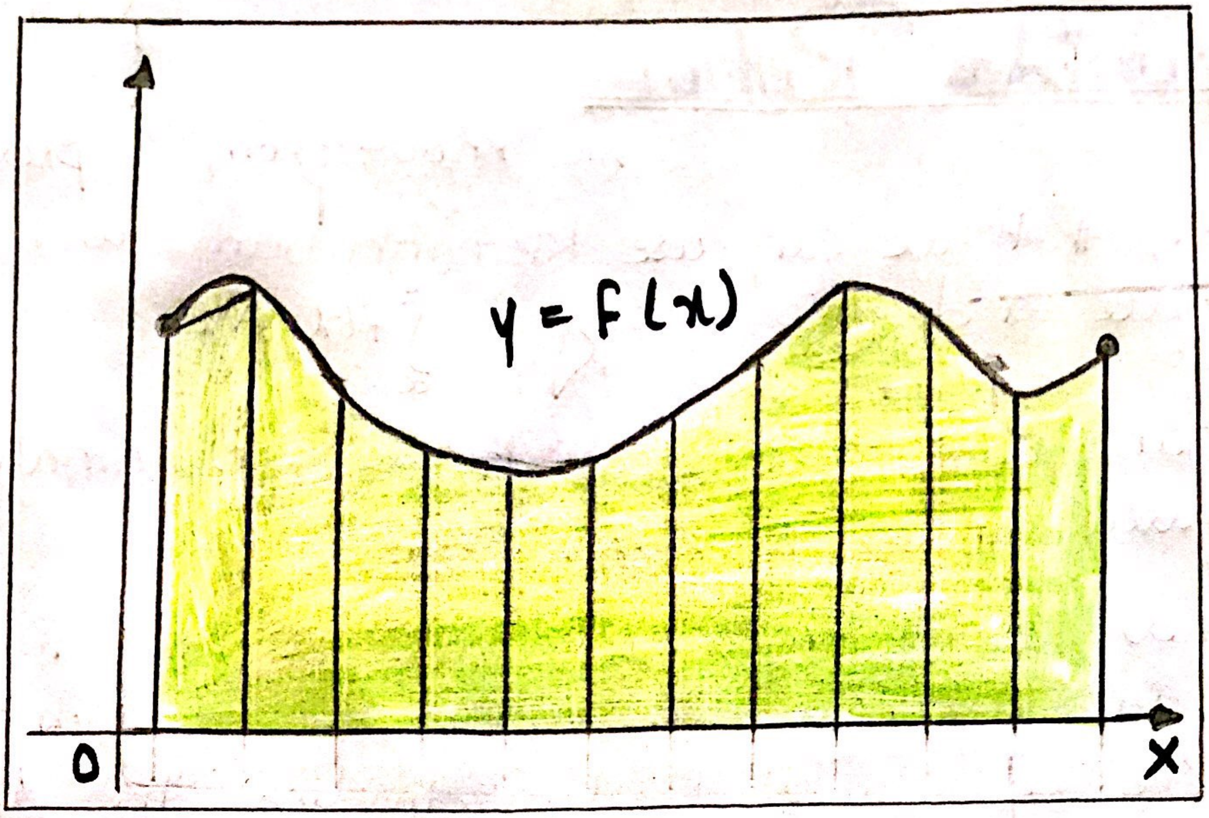
Another useful integration rule is the Trapezoidal Rule. Under this rule, the area under a curve is evaluated by dividing the total area into little trapezoids rather than rectangles.

Let $f(x)$ be continuous on $[a,b]$. We partition the interval $[a,b]$ into n equal subintervals, each of width.

$$\Delta x = \frac{b-a}{n},$$

Such that.

$$a = x_0 < x_1 < x_2 < \ldots\ldots < x_n = b.$$



The Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is given by.

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\left[f(x_0) + 2(x_1) + 2f(x_2) \cdots + 2f(x_{n-1}) + f(x_n)\right],$$

where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\,\Delta x$.

As $n \rightarrow \infty$, the right-hand side of the expression approaches the definite intergeal $\int_a^b f(x)\,dx$.

Simpson's Rule

Simpson's Rule is a numerical method that approximates the value of a definite integral by using quadratic functions.

The method is named after the English mathematician Thomas Simpson. (1710-1761).

Simpson's Rule is based on the fact that given three points, we can find the equation of a quadratic through those points.

To obtain an approximation of the definite integral $\int_a^b f(x)\,dx$ using Simpson's Rule, we partition the interval $[a,b]$ into an even number n of subintervals, each of width.

$$\Delta x = \frac{b-a}{n}$$

One each pair of consecutive subintervals $[x_{i-1}, x_i]$, $[x_i, x_{i+1}]$, we consider a quadratic function $y = ax^2 + bx + c$ such that it passes through the points $(x_{i-1}, f(x_{i-1}))$, $(x_i, f)(x_i))$, $(x_{i+1}, f(x_{i+1}))$.

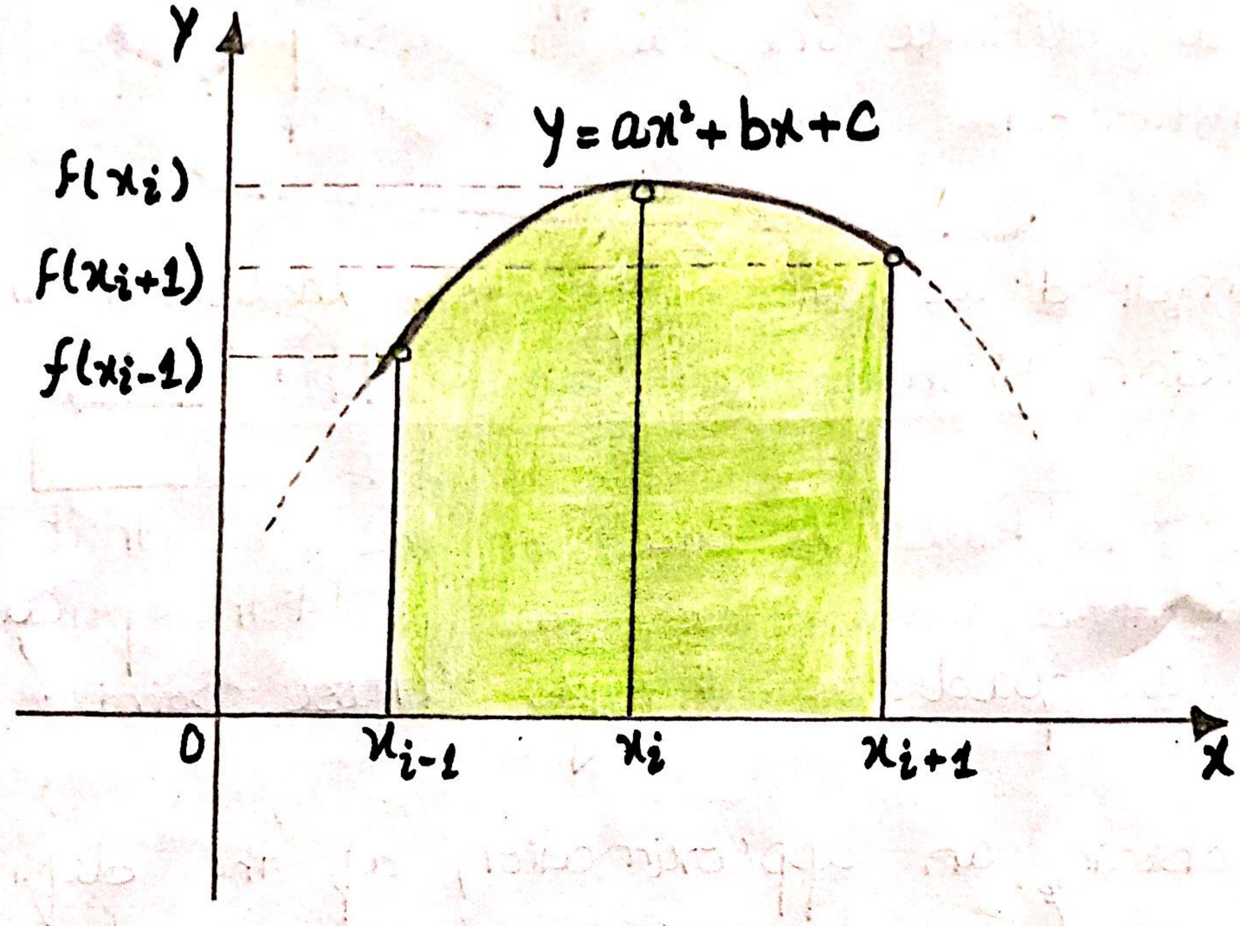


Figure 1.

If the function $f(x)$ is continuous on $[a, b]$, then

~~the~~ $$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)\right.$$

$$+ 4f(x_3) + 2f(x_4) + \cdots\cdots + 4f(x_{n-1}) + f(x_n)\Big].$$

The coefficients in Simpson's Rule have the following pattern.

$$\underbrace{1, 4, 2, 4, 2 \cdots\cdots, 4, 2, 4, 1.}_{n+1 \text{ points}}$$