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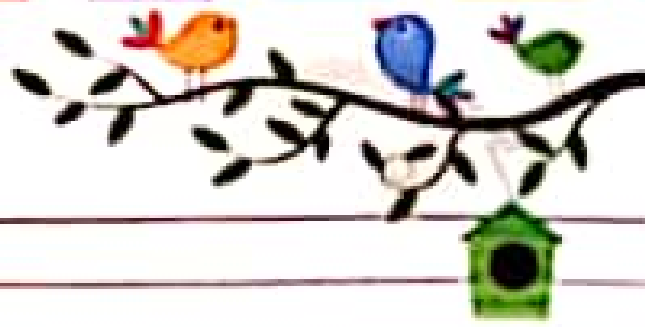
Subject :- LINEAR ALGEBRA

Department :- Be. Civil

Student Sign :- T Hussain<sup>\*</sup>

Semester :- 12<sup>th</sup>.

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Question 1:-

$$x_1 - x_2 + x_3 = 0$$

$$2x_2 - 8x_3 = 8$$

$$5x_1 - 5x_2 = 10$$

Sol:-

To find  $x = ?$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 0 & 5 & -10 \end{bmatrix}$$

$R_3 + (-5/R_2)$



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$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -8 \\ 0 & 1 & -2 \end{bmatrix}$$

$R_3 \times \frac{1}{2}$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -2 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 - R_2$

Rank of  $A = 3$



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Now Rank of AB

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 5 & -10 & 10 \end{array} \right] \quad R_3 - 5R_1$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 0 & -5 & 10 \end{array} \right] \quad R_3 + 5R_1$$

So Rank of AB = 3

So the Question is

Consistent



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Question 2:-

Find the inverse of  $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$   
by adjoint Method

Soln

To find  $|A| = ?$

$$|A| = 3 \begin{vmatrix} -1 & 7 \\ -2 & 7 \end{vmatrix} - 4 \begin{vmatrix} 2 & 7 \\ 5 & 7 \end{vmatrix} + 5 \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$\begin{aligned} |A| &= 3(-7 + 14) - 4(14 - 35) + 5(-4 + 5) \\ &= 3(7) - 4(-21) + 5 \\ &= 21 + 84 + 5 \end{aligned}$$

$$|A| = 110$$

Now to find adjoint of  $A = ?$

$$\text{adj } A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

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$$a_{11} = (-1)^{11} (-7+14) = (-1)^9 (7) = 7$$

$$a_{22} = (-1)^{11} (14-21) = (-1)(-7) = 7$$

$$a_{33} = (-1)^4 (-4+2) = 1$$

$$a_{21} = (-1)^8 (28-10) = 18$$

$$a_{32} = (-1)^4 (21-25) = (-1)(-4) = 4$$

$$a_{23} = (-1)^5 (-6-20) = (-1)(-26) = 26$$

$$a_{31} = (-1)^4 (28+15) = 43$$

$$a_{32} = (-1)^5 (21-10) = -11$$

$$a_{23} = (-1)^6 (-3-8) = -11$$

Now

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{110} \begin{bmatrix} 7 & -38 & 43 \\ 21 & -4 & -11 \\ 1 & 26 & -11 \end{bmatrix}$$

Ans

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Question 5:-

$$2x + 2y + 4z = 18$$

$$x + 3y + 2z = 13$$

$$3x + 2y - 3z = 14$$

Write in Matrix form

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & 13 \end{bmatrix} \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

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$$\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & 13 \end{array} \right] \quad \begin{array}{l} R_1 \\ \hline 2 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & 15 \end{array} \right] \quad \begin{array}{l} R_1 - R_2 \\ R_3 + R_2 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -15/9 \end{array} \right] \quad \begin{array}{l} R_3 \\ \hline 3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 31/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -15/9 \end{array} \right] \quad \begin{array}{l} R_1 + 7(-3R_3) \end{array}$$



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Solve by Gauss Jordan Method

$$\frac{R_1}{c} \quad x = \frac{8}{3}$$

$$y = 2$$

$$z = -\frac{5}{3}$$

~~Answer~~

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Question 5:

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-3x_1 - 25x_2 + 4x_3 = 0$$

$$6x_1 + x_2 - 8x_3 = 0$$

Sol:

To find  $\Delta$

$$\begin{vmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{vmatrix} = 3(200 - 4) - 0 - 4(24) = 0$$

So the  $\Delta = 0$

It means it is non trivial sol.

$$\Delta = \begin{vmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{vmatrix}$$

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$$\left[ \begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array} \right] \quad R_2 \times 1/3$$

$$\left[ \begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & -49 & 0 & 0 \end{array} \right] \quad \begin{array}{l} R_2 + 3R_1 \\ R_3 + 2R_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right] \quad \begin{array}{l} R_2 \leftrightarrow R_3 \\ R_2 \times (-1/20) \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -20 & 0 & 0 \end{array} \right] \quad R_2 \leftrightarrow R_3$$

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$$\begin{array}{ccc|c} 1 & 5/3 & -4/3 & \\ \hline 0 & 1 & 0 & R_3 + 20R_2 \\ \hline 0 & 0 & 0 & \end{array}$$

$\rightarrow$

$$R_2 \Rightarrow x_2 = t$$

$$R_3 \Rightarrow x_3 = t$$

$$R_1 \Rightarrow x_1 + 5/3 x_2 - 4/3 x_3 = 0$$

$$x_1 + 5/3 t - 4/3 t = 0$$

$$x_1 + t \left( \frac{5-4}{3} \right) = 0$$

$$x_1 + \frac{1}{3} t = 0$$

$$x_1 = -\frac{1}{3} t$$

$\rightarrow$

$$\begin{array}{l} x_1 = -\frac{1}{3} t \\ x_2 = t \\ x_3 = t \end{array}$$

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Question 6:- Reduce the matrix to normal form and find rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 4 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{array}{l} R_2 + (-3R_1) \\ R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_2 \leftrightarrow R_3$$

$$\begin{bmatrix} 1 & 3 & 4 & 0 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_1 + R_2$$

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$$\begin{bmatrix} 1 & 3 & 4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{R_2}{13}$$

So the rank is

$$\boxed{R = 3}$$

Answer