QUESTION # 1 (a)

a) Calculate the Correlation Co-effiecent between x and y.

x	y	xy	x^2	y^2
3	25	75	9	625
4	24	96	16	576
5	20	100	25	400
6	20	120	36	400
7	19	133	47	361
8	17	136	64	289
9	16	144	81	256
10	13	130	100	169
11	10	110	121	100
13	8	104	169	64
$\Sigma x = 76$	$\Sigma y = 172$	$\Sigma xy = 1148$	$\Sigma x^2 = 668$	$\Sigma y^2 = 3240$

Formula:

$$r_{xy} = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

$$r_{xy} = \frac{1148 - \frac{(76)(172)}{10}}{\sqrt{\left\{(668) - \frac{(76)^2}{10}\right)\left((3240)^2 - \frac{(172)^2}{10}\right)}}$$

$$r_{xy} = \frac{1148 - 1307.2}{\sqrt{(668 - 577.6)(3240 - 2958.4)}}$$

$$r_{xy} = \frac{-159.2}{\sqrt{(90.4) \cdot (281.6)}} = \frac{-159.2}{\sqrt{25456.64}}$$

$$r_{xy} = \frac{-159.2}{159.6}$$

$$\boxed{r_{xy} = -0.997}$$

QUESTION # 1 (B)(a)

X	Y	X^2	Y^2	XY
20	5	400	25	100
11	15	121	225	165
15	14	225	196	210
10	17	100	289	170
17	8	289	64	136
18	9	324	81	162
21	12	441	144	252
25	16	625	256	400
28	18	784	324	504
$\Sigma X = 165$	$\Sigma Y = 114$	$\Sigma X^2 = 3309$	$\Sigma Y^2 = 1604$	$\Sigma XY = 2099$

$$\hat{Y} = a + bx$$

where $b = \dfrac{n \Sigma XY - (\Sigma x)(\Sigma Y)}{n \Sigma x^2 - (\Sigma x)^2}$

$$b = \frac{(9)(2099) - (165)(114)}{(9)(3309) - (165)^2}$$

$$b = \frac{18891 - 18810}{29781 - 27225}$$

$$b = \frac{81}{2556}$$

$$b = 0.03$$

$a = \bar{Y} - b\bar{X}$ $\bar{Y} = \frac{\Sigma Y}{n}$

$\bar{X} = \frac{\Sigma X}{n}$

Putting Values.

$\bar{Y} = \frac{114}{9}$

$\bar{Y} = 12.67$

$\bar{X} = \frac{165}{9}$

$\bar{X} = 18.33$

$a = \bar{Y} - b\bar{x}$

$a = 12.67 - (0.03)(18.33)$

$a = 12.67 - 0.5499$

$a = 12.12$

Thus Co-efficient of regression line is

$\hat{Y} = a + bx$

$\hat{Y} = 12.12 + 0.03\,X$

Now,

X on Y

$$\hat{X} = a_0 + b_0 Y$$

where

$$b_0 = \frac{n \sum XY - (\sum X)(\sum Y)}{n \sum Y^2 - (\sum Y)^2}$$

$$b_0 = \frac{9(2099) - (165)(114)}{9(1604) - (114)^2}$$

$$b_0 = \frac{18891 - 18810}{14436 - 12996}$$

$$b_0 = \frac{81}{1440}$$

$$b_0 = 0.056$$

$$a_0 = \bar{X} - b_0 \bar{Y}$$

$$a^0 = \bar{X} - b_0 \bar{Y}$$

$$\bar{X} = \frac{\sum X}{n} = \frac{165}{9}$$

$$\bar{X} = 18.33$$

$$\bar{Y} = \frac{\sum Y}{9} = \frac{114}{9}$$

$$\bar{Y} = 12.67$$

Scanned with CamScanner

$a_0 = \bar{X} - b_0 \bar{Y}$

$a_0 = 18.33 - 0.056 \, (12.67)$

$a_0 = 18.33 - 0.70952$

$a_0 = 17.62$

Thus the desired least square regression of X on Y is :

$\hat{X} = 17.620 + 0.056 \hat{Y}$

(B) The Predicted value of Y are found by substituting the values in the estimated equation

Thus for $x = 20$

$\hat{Y} = 12.12 + 0.03 \, (20)$

$\hat{Y} = 12.12 + 0.6$

$\hat{Y} = 12.72$

Similarly for 11, 15, 25, 28.

$\hat{Y} = 12.12 + 0.03 \, (11) = 12.45$

$\hat{Y} = 12.12 + 0.03 \, (15) = 12.57$

$\hat{Y} = 12.12 + 0.03 \, (25) = 12.87$

$\hat{Y} = 12.12 + 0.03\ (28) = 12.96$

Now, x for Y

Finding (5)

$\hat{x} = 17.620 + 0.056\ (5) = 17.9$

For 15

$\hat{x} = 17.620 + 0.05\ (15) = 18.46$

For 9

$\hat{x} = 17.620 + 0.05(9) = 18.124$

For 12

$\hat{x} = 17.620 + 0.05(12) = 18.292$

For 16

$\hat{x} = 17.620 + 0.05\ (16) = 18.516$

For 18

$\hat{x} = 17.620 + 0.05\ (18) = 18.628$

QUESTION # 3

(a) UNGROUPED FREQUENCY DISTRIBUTION

CLASS	FREQUENCY	TALLY	RELATIVE FREQUENCY
0	1	1	$\frac{1}{50} \times 100 = 2\%$
1	4	1111	$\frac{4}{50} \times 100 = 8\%$
2	8	~~1111~~ 111	$\frac{8}{50} \times 100 = 16\%$
3	11	~~1111~~ ~~1111~~ 1	$\frac{11}{50} \times 100 = 22\%$
4	8	~~1111~~ 111	$\frac{8}{50} \times 100 = 16\%$
5	5	~~1111~~	$\frac{5}{50} \times 100 = 10\%$
6	4	1111	$\frac{4}{50} \times 100 = 8\%$
7	3	111	$\frac{3}{50} \times 100 = 6\%$
8	2	11	$\frac{2}{50} \times 100 = 4\%$
9	1	1	$\frac{1}{50} \times 100 = 2\%$
10	3	111	$\frac{3}{50} \times 100 = 6\%$

(B)

CLASS LIMIT	(F)	TALLY	RELATIVE FREQUENCY
0 - 3	24	~~1111~~ ~~1111~~ ~~1111~~ ~~1111~~ 1111	$\frac{24}{50} \times 100 = 48\%$
4 - 6	17	~~1111~~ ~~1111~~ ~~1111~~ 11	$\frac{17}{50} \times 100 = 34\%$
7 - 10	9	~~1111~~ 1111	$\frac{9}{50} \times 100 = 18\%$
11 - 13	0	0	$\frac{0}{50} \times 100 = 0\%$

QUESTION # 2(a)

Solution :-

$$n = 5$$

$$P = \frac{1}{2} \quad ; \quad q = 1/2$$

$$\text{Mean} = \bar{x} = np$$

$$\text{Standard deviation} = npq$$

$$P(X = \text{No. of heads}) = ?$$

$$P(X = x) = ?$$

Using bionomial formula :

$$P(X = x) = \binom{n}{x} P^{x} \cdot q^{n-x}$$

$$P(x = 0) = \binom{5}{0} \left(1/2\right)^{0} \left(1/2\right)^{5}$$

$$P(x = 0) = \cancel{0.156}\ 0.0312$$

$$P(x = 1) = \binom{5}{1} \left(1/2\right)^{1} \left(1/2\right)^{4}$$

$$P(x = 1) = 0.156$$

$$P(x = 2) = \binom{5}{2} \cdot \left(1/2\right)^{2} \left(1/2\right)^{3}$$

$$P(x = 2) = 0.3125$$

$P(x=3) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

$P(x=3) = 0.3125$

$P(x=4) = \binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1$

$P(x=4) = 0.156$

$P(x=5) = \binom{5}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

$P(x=5) = 0.03125$

Now to find np and npq

We have to find p and q

$n = 5$, $p = \frac{1}{2}$ $q = \frac{1}{2}$

$\bar{X} = np = 5 \times \frac{1}{2} = \frac{5}{2} = 2.5$

$\boxed{\bar{X} = 2.5}$

$S.D = \sqrt{npq}$

$S.D = \sqrt{5 \times \frac{1}{2} \times \frac{1}{2}}$

11

$$S.D = \sqrt{\frac{5}{4}}$$

$$S.D = \sqrt{1.25}$$

$$\boxed{S.D = 1.118}$$

(B)

Solution :-

Total Games = 10

P(A will win) = 2/3

(i) P(A will win at 4 games) = P (x ≥ 4) = ?

So, P(x ≥4) = 1 - P (x < 4)

$$P(x \geq 4) = 1 - \left[\binom{10}{0}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{0} + \binom{10}{1}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{1} + \binom{10}{2}\left(\frac{2}{3}\right)^{8}\left(\frac{1}{3}\right)^{2} + \binom{10}{3}\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right)^{3} \right]$$

$$P(x \geq 4) = 1 - [0.0173 + 0.0867 + 0.1950 + 0.2601]$$

$$P(x > 4) = 1 - [0.5591]$$

$$P(x \geq 4) = 0.4409$$

(ii) P(Exactly equal to $\frac{4}{10}$ games) = ?

$$P\left(x = \frac{4}{10}\right) = 0$$

As The probability of fractional value is either impossible or zero.

(iii) P (Exactly equal to 11 games) =

It is not possible as total no. of games are 10. So he cannot win 11 games.

(iv) P(6 or more games) =

$$= \binom{10}{6}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^6 + \binom{10}{7}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^7$$

$$+ \binom{10}{8}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^8 + \binom{10}{9}\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^9$$

$$+ \binom{10}{10}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^{10}$$

P(6 or more games) = 0.056 + 0.016 + 0.003 + 0.0003 + 0.00001

P(6 or more games) = **0.07531**