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Section : B.

Subject : Differential Equations.

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Q No. 1:-

Objective type Questions.

1:- AB order = $m \times n$

2:- 1 Non-zero row

3:- $a = 8$

4:- $(2ix - i) - i^2 = 2i^2 - i^2 = -3(-1) = \boxed{3}$

5:- Scalar matrix.

6:- $\log y = x - x^2 + c$
or $y = e^{x - x^2} + c$.

7:- order = 1, degree = 3.

8:- order degree is not defined as it's not a polynomial.

9:- Homogenous equation.

10:- $(a-b)(b-c)(c-a)$.

Q = 2:-

A :- Express the determine:-

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$$

Sol

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$$

as the product factor in which the linear in a.b.c

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$$

Expand by R1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(a^2b^2 - b^3c^3) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^2b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

$\because$ common abc.

$$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc[bc(c-b) - ac(c+a) + ab(b-a))$$

Ans:

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Q No = 21 -

3

B :- $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

by Eigen value.

Sol :-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic eq $\rightarrow |A - \lambda I| = 0 \rightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determent.

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0 \rightarrow (B)$$

Expand by R_1 (4)

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow (B)$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \text{ Expand by } R_1$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda) - (-1)(-1)\big) - 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow (a)$$

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$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow (b)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow (c)$$

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Put a, (b) & (c) in (B) (6)

$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

$= 2\lambda^3+16\lambda^2-36\lambda+\lambda^2-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8$
$-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$
$+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By synthatic division

we get.

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$(\lambda=0)$

$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$

$\lambda^2-8\lambda+16=0$

By factrization method

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)$.

$\lambda=4, \lambda=4$.

$\boxed{\lambda_1=0, \lambda_2=2, \lambda_3=4. \lambda_4=4}$

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Q = 3 :-

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$x = 2 \ , \ y = 6.$$

Sol :-

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$$

Divide b|side by $2xydx$

we get.

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2x\cancel{y}} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow Z$$

Let $y = vx$

Diff $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \longrightarrow (a)$$ (8)

Put (a) in (2)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

x ing b/side by "2"

$$2v + \frac{2x\,dv}{dx} = \frac{1}{v} + 3v$$

$$\frac{2x\,dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$\frac{2x\,dv}{dx} = \frac{1}{v} + v$$

$$\frac{2x\,dv}{dx} = \frac{1+v^2}{v}$$

x ing b/side by $\frac{dx}{dv}$

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

xing b/side by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on b/side

$$\int \frac{2v}{1+v^2}\,dv = \ln x + \ln c$$

take "e" on b/side

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

Put $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \rightarrow (22)$$

Put $x=2, y=6$ in eq (22)

$(4)+(36) = 8c$

$c = \frac{40}{8}$

$c = 5 \rightarrow$ Put in (22)

So

$x^2 + y^2 = 5x^2$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking "$\sqrt{\ }$" on b/side.

$y = \pm x\sqrt{5x-1}$

$\{y = -x\sqrt{5x-1}\}$

or $\{y = \pm x\sqrt{5x-1}\}$ Ans.