IQRA NATIONAL UNIVERSITY

Digital Signal Processing

Final Assignment Spring 2020

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Question 1-

(A) Part :-

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \longrightarrow (1)$$

The homogenous equation of the System

$$y(n) - 3y(n-1) - 4y(n-1) = 0$$

The charastic equation of the System.

$$\cancel{y(n)} \quad \lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\lambda = 4 \quad , \quad \lambda = -1$$

So,

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$y_h(n) = C_1(-1)^n u(n) + C_2(4)^n u(n)$

Since 4 is a characteristic root and the excitation is;

$$x[n] = 4^n u[n]$$

Assume a particular solution.

$$y_p[n] = kn\, 4^n u[n]$$

Then, $kn4^n u[n] - 3k[n-1]4^{n-1}u[n-1] - 4k[n-2]4^{n-2}$

$u[n-2] = 4^n u(n) + 2(4)^{n-1} u(n-1)$

For $n=2$;

$$k(32-12) = 4^2 + 8 = 24$$

$$k = 6/5$$

The total solution is;

$$y(n) = y_p(n) + y_h(n)$$

$$\left[\tfrac{6}{5} n4^n + C_1 4^n + C_2(-1)^n\right] u[n]$$

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The solve for C_1 & C_2 we assume that;

$$y(-1) = y(-2) = 0$$

$$y(0) = 1 \quad \& \quad y(1) = 3y(0) + 4 + 2 = 9$$

Hence;

$$C_1 + C_2 = 1$$

$$24/5 + 4C_1 - C_2 = 9$$

$$4C_1 - C_2 = 21/5$$

$$C_1 = 26/25$$

$$C_2 = -1/25$$

The total solution is

$$y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4n - \frac{1}{25}(-1)^n\right] u[n]$$

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Question - 1
(B) Part :-
Solution :-

$$y[n] = 0.6y(n-1) - 0.08y[n-2] + x[n]$$

Homogenous equation;

$$x[n] = 0$$

$$y[n] = 0.6y[n-1) + 0.08y(n-1) = 0$$

$$y_h(n) = \lambda^n$$

So,

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$(\lambda - 0.2)(\lambda - 0.8) = 0$$

$$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$$

Thus, the general form of the solution to the homogenous equation is;

$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$

$y(n) = C_1(0.2)^n + C_2(0.4)^n \longrightarrow (1)$

$y_h(n) = C_1 \frac{1}{5}^n + C_2\left(\frac{2}{5}\right)^n$

$y(0) = 1 \quad , \quad y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

$\frac{1}{5}C1 + \frac{2}{5} = 0.6$

$C1 = -1 \quad , \quad C_2 = 3$

Therefore,

$h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]U[n]$

The step response;

$y(n) = \sum_{k=0}^{n} h(n-k) \qquad n \geq 0$

$y(n) = \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$

$= \frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\frac{1}{5}^{n+1} - 1\right]U(n)$

Question - 2

(A) Part:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution:-

Taking inverse z-transform.

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence,

$$x(n) = [4(2)^n - 3 - n]\, u(n)$$

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Question - 2

(B) Part:

$$X(z) = \frac{1}{1 - az^{-1}}$$

$$|z| > |a|$$

Using the complex inversion integral.

Solution:- We have,

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z - a}$$

where c is circle at radius greater then $|a|$. we shall evaluate this integral Using (3,4,2) with $f(z) = z^n$.

we distinguish two cases.

1- If $n \geq 0$, $f(z)$ has at radius only zeros and hence no poles inside c. The only pole inside c is $z = a$.

Hence,

$$x(n) = f(z_0) = a^n \qquad n \geq 0$$

2- If $n<0$ $f(z) = z^n$ has an nth-order pole at $z=0$, which is also inside C. Thus there are contributions from both poles. For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n = -2$, we have

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n<0$, Thus

$$x(n) = a^n u(n)$$

Question - 3

(A) part : $H(z) = \frac{b_0}{(1-pz^{-1})^2}$

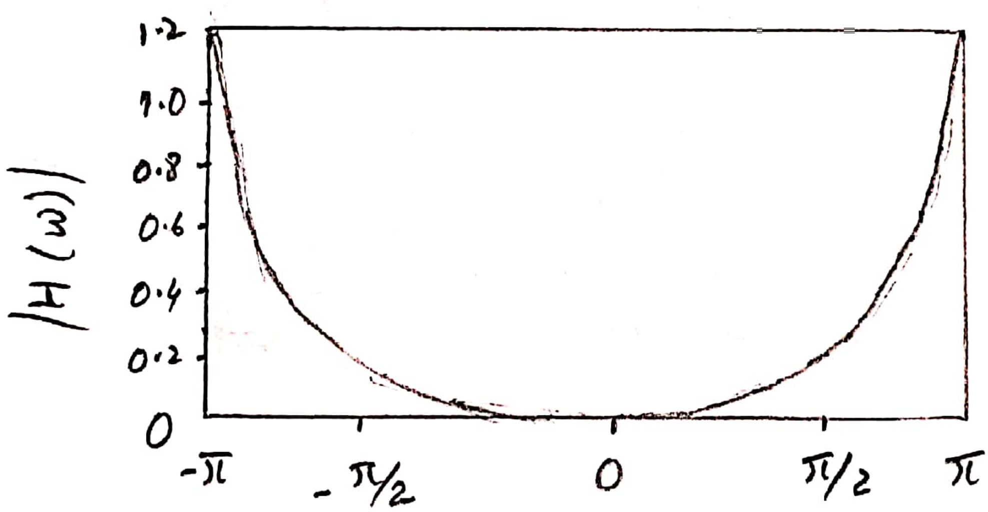
$H(0) = 1$ & $|H(\pi/4)|^2 = 1/2$

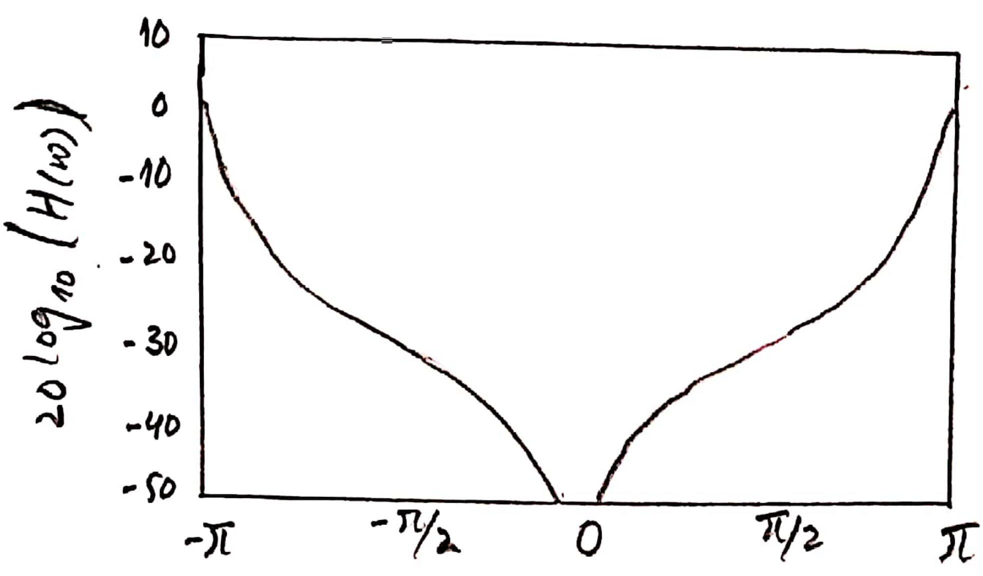
Solution :-

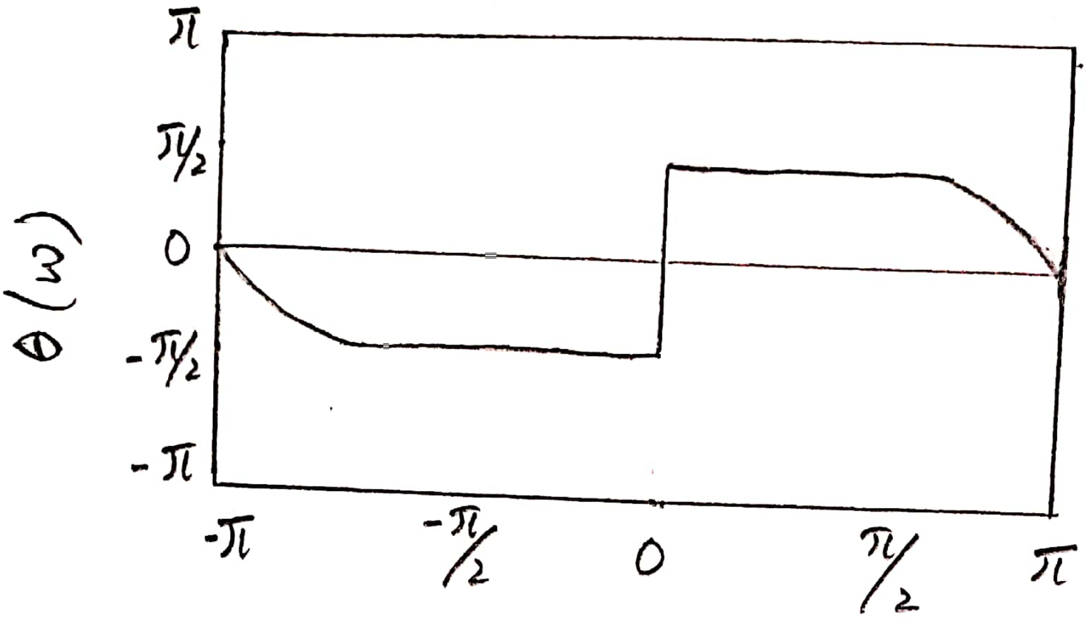
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence, $b_0 = (1-p)^2$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{\left(1-Pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-P)^2}{\left[1-P\cos(\pi/4)+ jP\sin(\pi/4)^2\right]}$$

$$= \frac{(1-P)^2}{\left(1-P/\sqrt{2}+jP/\sqrt{2}\right)^2}$$

Hence,

$$= \frac{(1-P)^4}{\left[(1-P/\sqrt{2})^2 + P^2/2\right]^2}$$

$$= 1/2$$

Equivalently,

$$\sqrt{2}\,(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

⇒ The system function for desired filter,

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Question - 3

(B) part :-

Solution :- The filter must have poles at,

$$p_{12} = re$$

And zero at $z = 1$ And $z = -1$.

consequently. the same system funtion is,

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$H(z) = G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$.

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$.

we have;

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4} \cdot \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

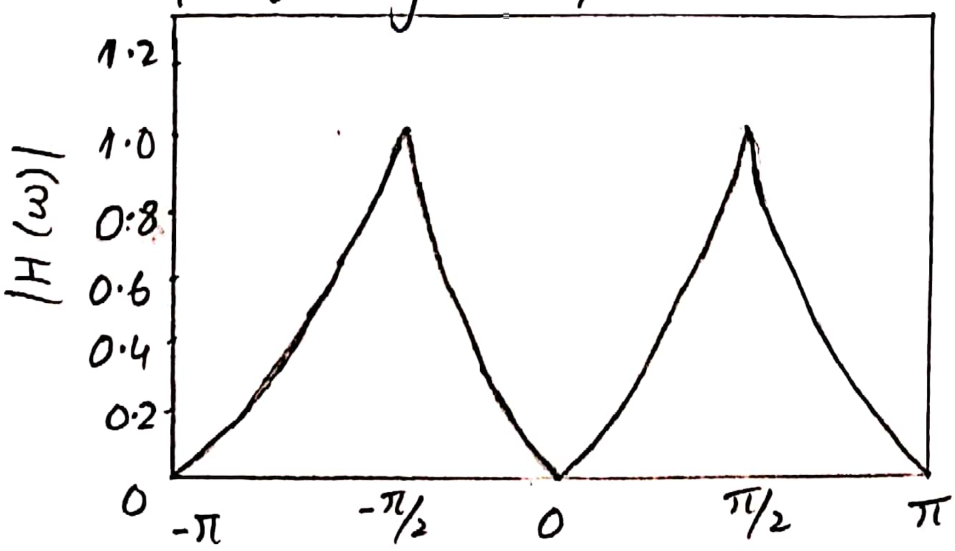
Equivalently;

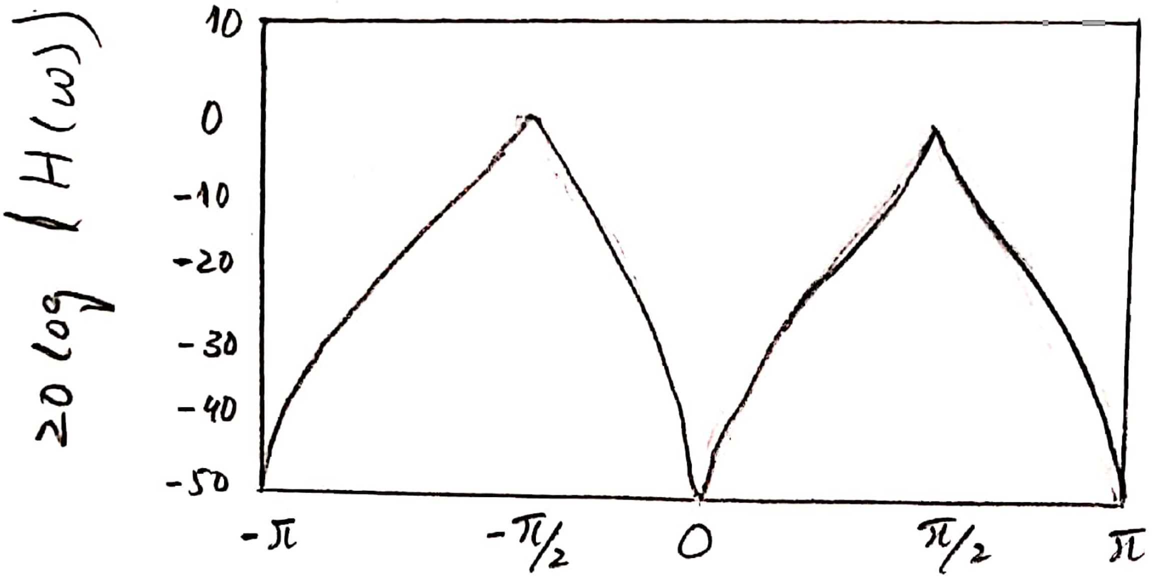
$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

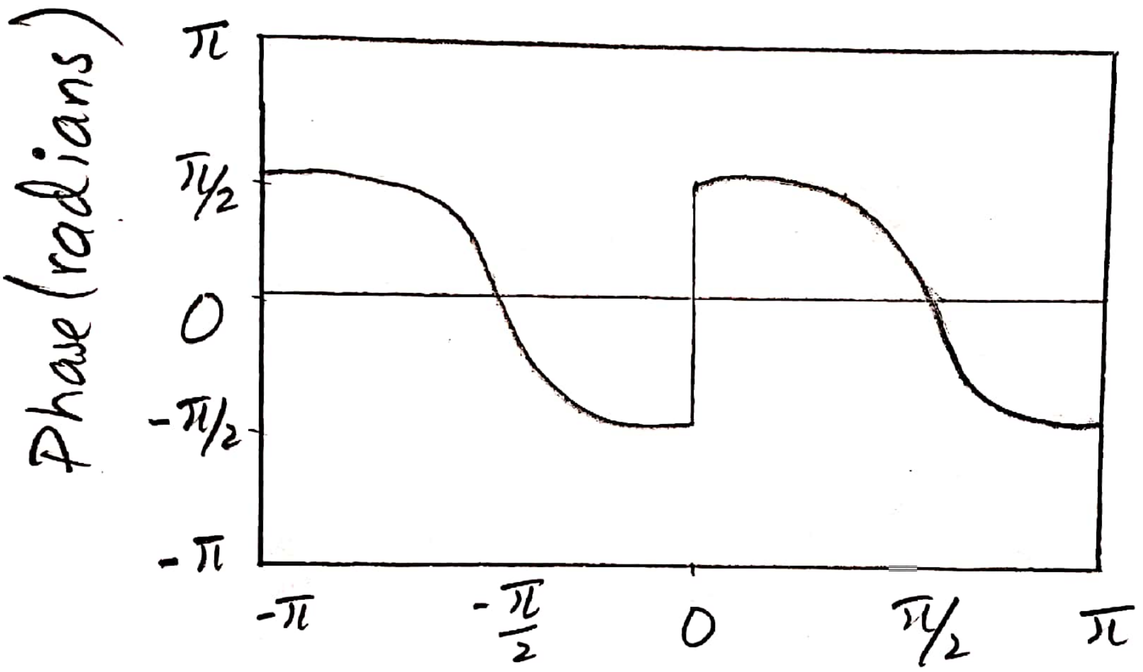
The value of $r^2 = 0.7$ satisfies this equation. Therefore, the System function for the desired filter is;

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its frequency response is illustrated







Magnitude and phase response of a simple bandpass filter is,

$$H(z) = 0.15\left[\left(1-z^{-2}\right) / \left(1+0.7\, z^{-2}\right)\right].$$

Question - 4

(A) Part:

$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Solution:- The Fourier transform of this sequance is;

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$X(\omega) = \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}.$$

The magnitude and phase of $X(\omega)$ are illustrated for $L=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies

$$\omega_k = 2\pi k/N,$$

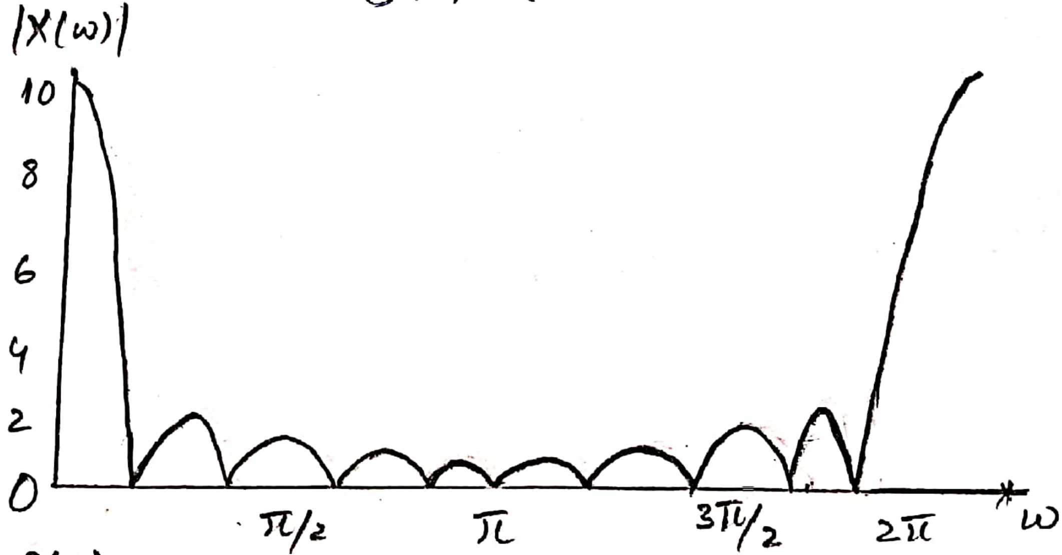
$$k = 0, 1, 2, \ldots\ldots N-1$$

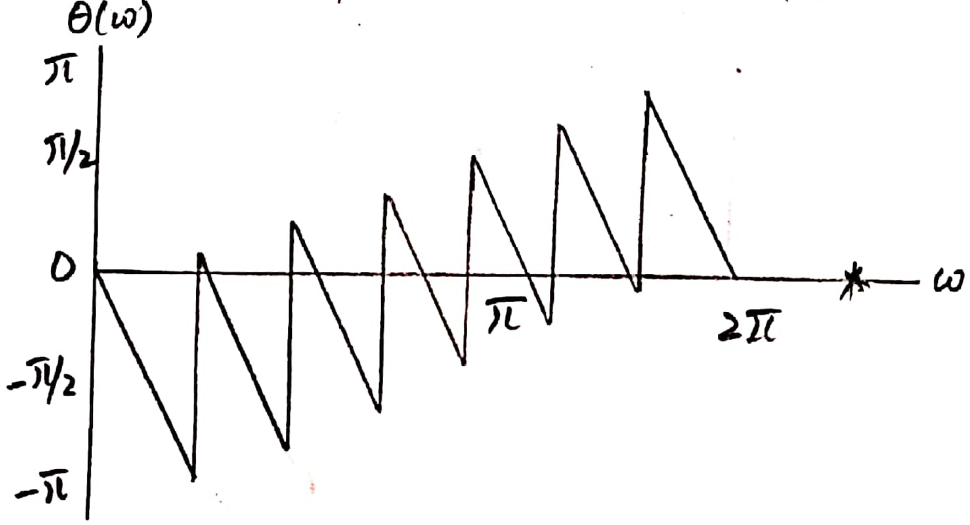
Hence,

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}}$$

$$k = 0, 1, \ldots\ldots, N-1$$

$$X(k) = \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)N}$$





If N is selected such that $N = L$ then the Discrete Fourier Transform becoms,

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2, \ldots L-1 \end{cases}$$

Thus, there is only one nonzero value in the DFT, This is apparent from observation of $X(\omega)$, since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k / L$

$k \neq 0$;

The reader should verify that $x(n)$ can be recoverd from $X(k)$ by performing an L-point IDFT.

Provides a plot of the N-point DFT, in magnitude and phase, for $L = 10$, $N = 50$ and $N = 100$. Now the spectral characteristics of the sequence are more clearly evident.

Question - 4
(B) part :-

Solution:-
The first step is to determine the matrix W_4. By exploiting the periodicity property of W_4.

$$W_N^{K+N/2} = -W_N^K$$

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^{02} & W_4^{22} & W_4^2 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^3 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

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They

$$Y_4 = W_4 X_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of X_4 may be determined by conjugating the element in between to obtain W_4^+ & then applying the formula.