Final Paper:

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Sec: B

Subject: Differential Equation.

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Que: no 1

The Wave Equation:

The wave equation is an important second-order linear partial differential equation. for the description of wave as they occur in classical physic - such as mechanial wave (e.g water waves, sound wave and sesmic wave.

➤ We generally visit beach and if we stand on an ocean shore and take a snapshots of the wave . . . . by 1-Dimensional equation.

$$\frac{\partial^2 w}{\partial t} = c^2 \frac{\partial^2 w}{\partial x^2}.$$

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

i- $w = \sin(x+ct) + \cos(2x+2ct)$

ii- $w = \tan(2x+ct)$.

i $w = \sin(x+ct) + \cos(2x+2ct)$.

Solution:

$w = \sin(x+ct) + \cos(2x+2ct)$.

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2$$

(1)

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2.$$

$$\frac{\partial w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

$$c^2 \cdot \frac{\partial^2 w}{\partial x^2}.$$

Hence.

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

ii- $w = \tan(2x+ct)$.

Solution: →

$w = \tan(2x+ct)$.

A: we know that.

$w = \tan(2x+ct)$

Taking derivate w.r.t.x.

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t} \tan(2x+ct) \cdot c$$

→ $$\frac{\partial w}{\partial t} = \cancel{\partial} \sec^2(2x+ct) \cdot c$$

$$\frac{\partial w}{\partial t} = c \sec^2(2x+ct)$$

Again Taking Derivate.

$$\frac{\partial^2 w}{\partial t^2} = c \frac{\partial}{\partial t} \sec^2 (2x + ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c \cdot 2 \sec(2x+ct) - \sec(2x+ct) \times \tan(2x+ct) c$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2 \sec^2(2x+ct) - \tan(2x+ct)$$

Now $w = \tan(2x+ct)$

Taking derivate w.r.t x.

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x} \tan 2x + ct.$$

$$\frac{\partial w}{\partial x} = \sec^2(2x+ct) \cdot 2$$

$$\frac{\partial w}{\partial x} = 2 \sec^2 (2x+ct).$$

→ Again taking derivate

$$\frac{\partial^2 w}{\partial x^2} = 2\frac{\partial}{\partial x} \sec^2(2x + ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2.2 \sec(2x+ct)\sec(2x+ct) \tan(2x+ct) \cdot 2$$

→ $$\frac{\partial^2 w}{\partial x^2} = 6 \sec^2(2x+ct)\tan(2x+ct)$$

As we know the wave Equation

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

put values in wave Equation

$$1 \neq 3$$

Hence

$$L.H.S \neq R.H.S$$

Hence it is not the proff wave equation.

Que: no 2

Expand the following Function in a Fourier Series.

$$F(x) = x, \quad -\pi < x \le 0$$
$$= 2x, \quad 0 \le x < \pi$$

$a_0 = ?$, $a_n = ?$, $b_n = ?$

Solution,

Now:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx.$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\Big|_{-\pi}^{0}\right] + \frac{2}{\pi}\left[\frac{x^2}{2}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^2}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2} \quad \text{---} \quad (1).$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\, dx$$

$$= \frac{-1}{\pi}\left[\pi\int_{-\pi}^{0}\left(x\cos nx\,dx\right) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx\right]$$

$$= \frac{-1}{\pi}\left[x\left(\frac{\sin nx}{x}\right) - \left(-\frac{\cos nx}{n^2}\right)\Big|_{-\pi}^{0}\right] +$$

$$\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\Big|_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2} + \frac{2}{\pi}\left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

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So

$$a_n = \begin{cases} \dfrac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & n \text{ is even} \end{cases} \quad -(2).$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx$$

$$+ \frac{2}{\pi}\int_{0}^{\pi} x\sin nx.$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\Big|_{-\pi}^{0}\right]$$

$$+ \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\Big|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}.$$

Hence.

$$F(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)}{(2n-1)^2} + 3 \sum_{n=1}^{\infty} \frac{\sin nx}{n}.$$

The Required Fourier is.

Que:no 3

Solve the Initial value problem.

$$y'' - 4y' + 13y = 8\sin 3x$$

$$y(0) = 1 \quad , \quad y'(0) = 2$$

Solution:

$$y'' - 4y' + 13y = 8\sin 3x \quad - - - - - (1)$$

Associated Homogenous Eq (1) is

$$y'' - 4y' + 13y = 0 \quad \text{———} \quad (2).$$

Into Auxiliary Equation.

put $y = m$ in (2)

$$m^2 - 4m + 13 = 0.$$

→ Use Quadratic Formula.

$a = 1, \quad b = -4, \quad c = 13.$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

$$m = \frac{4 \pm \sqrt{36}\,i}{2} = \frac{4 \pm 6i}{2}$$

$$m = 2 \pm 3i$$

$$m_1 = 2 + 3i, \quad m_2 = 2 - 3i.$$

$$y_c = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) \quad \text{(A)}.$$

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Let $y_p = A\cos 3x + B\sin 3x$ ——————— (A)

Differentiate w.r.t "x"

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

Again Differentiate w.r.t x

$$y_p'' = -9A\cos 3x + 3B\cos 3x$$

put in Eq (1).

$$(-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x)$$
$$+ 13(A\cos 3x + B\sin 3x) - 8\sin 3x$$
$$(-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$$
$$(4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$$

Comparing Coefficient.

$\sin 3x \Rightarrow \quad 4B + 12A = 8$ ------ (a).

$\cos 3x \Rightarrow \quad 4A - 12B = 0$

$4A = 12B \Rightarrow \boxed{A = 3B}$

put in * Eq a (*)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B \Rightarrow B = 1/5 \rightarrow$ (c).

put Equatin c in eq (*).

$A = \frac{3}{5}$ ---- (d).

put c and d in (*)

$y_p = \frac{B}{5} \cos 3x + \frac{1}{5} \sin 3x$ ---- (B)

The General Solution is:

$y = y_c + y_p$

$$y = e^{2x}\left(C_1 \cos 3x + C_2 \sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

Now we have to find the vaules of C_1 and C_2 for this.

put $x=0$ & $y=1$ in (C)

$$1 = e^{x(2)}\left(C_1 \cos 3(0) + C_2 \sin 3(0)\right) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0)$$

$$1 = \left(C_1(1) + C_2(0)\right) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = C_1 + \frac{3}{5}$$

$$C_1 = 2/5 \text{ ------ } \star 1$$

Differentiate C w.r.t x

$$y' = C_1\left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) + C_2\left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \text{ ------ } (D).$$

put $y' = 2$, $x = 0$ in (D)

$y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + C_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$

put $y' = 2$, $x = 0$

$2 = C_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) - 6/5 \sin 3(0) + \frac{3}{5}\cos 3(0)$

$2 = C_1(2) + C_2(3) - 0 + \frac{3}{5}$

$2 = 2C_1 + 3C_2 + \frac{3}{5}$

put $C_1 = 2/5$

$2 = \frac{4}{5} + 3C_2 + \frac{3}{5}$

$2 = \frac{7}{5} + 3C_2$

$3C_2 = 2 - \frac{7}{5} \Rightarrow C_2 = 3/15$. ---- $*_2$

put A_1 & A_2 in C

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$$

$$y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$$

The Required General Solution.

Que:no4

$(D^2 - DD')z = \cos x \cos 2y.$

Solution: $\longrightarrow$

$(D^2 - DD')z = \cos x \cos 2y.$

In CF is given by.

$$CF = \phi_1(y) + \phi_2(y+x)$$

while its PI is given by.

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

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The Complete Solution of the given PDE's

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y) \cdot \text{Ans.}$$