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# 16318

Dept.

BE(E)

Subject

linear Algebra

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Q1 Let  $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$  &  $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ 2 & 2 \end{bmatrix}$

(a)

Identify the (3,2) entry of AB.

Solution:- Identify (3,2) = ?

$$\text{Row}_3(A) \cdot \text{Column}_2(B)$$
$$\Rightarrow [0 \ 1 \ -2] \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \cancel{(0)}(4) \Rightarrow (0)(4) + (1)(-1) + (-2)(2)$$

$$\Rightarrow 0 + (-1) + (-4)$$

$$\Rightarrow -1 - 4$$

$$\Rightarrow \boxed{-5} \text{ Ans}$$

Q1 Label the quadratic polynomial  
 (b) that interpolate the points  
 (1,3), (2,4), (3,7)

Solution:-

$$\text{As } a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$

$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$

$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

$$\text{Now } (x_1, y_1) = (1, 3), \quad (x_2, y_2) = (2, 4)$$

$$(x_3, y_3) = (3, 7) \quad \text{Put in above}$$

$$a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 7$$

$$Ab = \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array} \right]$$

$$\underline{R} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array} \right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\underline{R} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array} \right] R_3 - 3R_2$$

$$\text{So } a_2 + a_1 + a_0 = 3 \quad \text{--- (1)}$$

$$-2a_1 - 3a_0 = -8 \quad \text{--- (2)}$$

$$\boxed{a_0 = 4} \quad \text{Put in (2)}$$

$$-2a_1 - 12 = -8 \Rightarrow a_1 = \frac{4}{-2} = -2$$

Put in (1)

$$a_2 - 2 + 4 = 3 \Rightarrow \boxed{a_2 = 1}$$



Q2 IF A & B are non matrices  
(a) where  $|A| = 2$  and  $|B| = -3$   
calculate  $|A^{-1} B^t|$

Solution!  $|A^{-1} B^t|$   
 $= |A^{-1}| |B^t|$

$$= \frac{1}{|A|} |B|$$

$$|B^t| = |B|$$

So  $|A^{-1} B^t| = \frac{1}{|A|} |B|$

$$= \frac{1}{2} \cdot 3$$

$$= \frac{3}{2} \text{ Ans}$$

Q2 Estimate the linear system of  
(b) equation

$$\begin{aligned}x + y + 2z &= 1 \\x - 2y + z &= -5 \\3x + y + z &= 3\end{aligned}$$

Solution:-

$$\begin{aligned}x + y + 2z &= 1 \\x - 2y + z &= -5 \\3x + y + z &= 3\end{aligned}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array} \right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array} \right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/3 & 2 \\ 0 & -2 & -5 & 0 \end{array} \right] \begin{array}{l} R_2 \\ -3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/3 & 2 \\ 0 & 0 & -13/2 & 4 \end{array} \right] R_3 + 2R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & 1 & -8/13 \end{array} \right] R_3 \times \frac{2}{-13}$$



$$x + y + 2z = 7 \quad \text{--- (i)}$$

$$y + \frac{7}{3}z = 2 \quad \text{--- (ii)}$$

$$z = \frac{-8}{13} \quad \text{(iii)}$$

Now Put eq (iii) in eq (ii)

$$y + \frac{7}{3} \times \frac{-8}{13} = 2$$

$$y - \frac{8}{39} = 2$$

$$y = 2 + \frac{8}{39}$$

$$y = \frac{78+8}{39} = \frac{86}{39}$$

Now put value of y in (i)

$$x + \frac{86}{39} + 2 \left( \frac{-8}{13} \right) = 1$$

$$x + \frac{86}{39} - \frac{16}{13} = 1$$

$$x + \frac{38}{39} = 1$$

$$x = 1 - \frac{38}{39}$$

$$\boxed{x = \frac{1}{39}} \quad \text{Ans}$$

Q3 Find  $A^{-1}$  where  $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Solution:-

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

Expand by Row 1

$$= 3 \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} - (-2) \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} + 2 \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$\Rightarrow 3(-18 - 0) + 2(-15 - 2) + 1(0 - 6)$$

$$\Rightarrow 3(-18) + 2(-17) + 1(-6)$$

$$\Rightarrow -54 - 34 - 6$$

$$\Rightarrow -94$$

$$|A| = -94$$



$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & -1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}^t$$

$$\Rightarrow \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix}$$