

Submitted By "Zasak Rehman"

Submitted To "Mam Shomaila Mazhar"

I.D 7666

Subject "Numerical Analysis"

Assignment No # 1



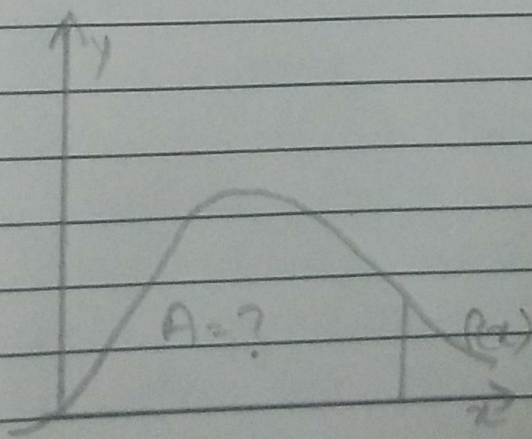
Ques 1:-

Review of integration Concept?

Introduction to Integration:-

Integration is a way of adding slices to find the whole.

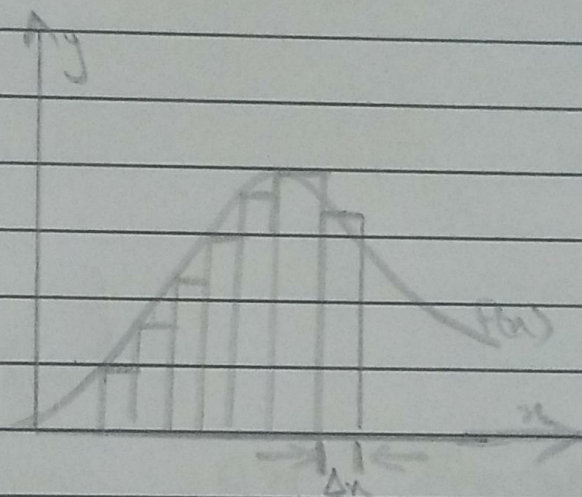
Integration can be used to find areas, volumes, central points and many useful things. But it is easiest to start with finding the area under the curve of a function like this.



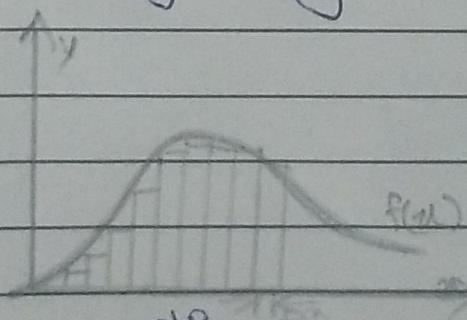
What is the area under  
 $y = f(x)$ ?



We could calculate the function at a few points and add up slices of width  $\Delta x$  like this (but the answer won't be very accurate).

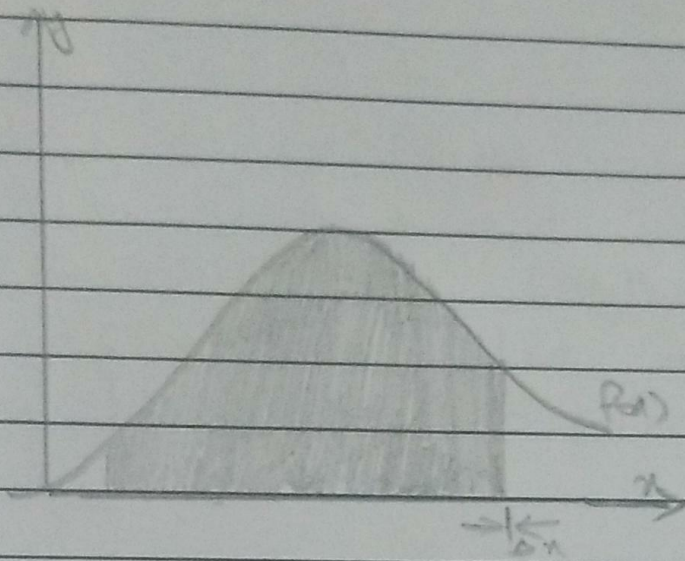


We can make  $\Delta x$  a lot smaller and add up many small slices (answer is getting better)



As the slices approach zero in width the answer approaches the true answer. we now write  $\Delta x$  to mean  $\Delta x$  slices are approaching zero.





## Question 2:-

Application of Trapezoidal rule and Simpson's rule in Engineering:

### Trapezoidal Rule:-

We know from previous lesson that we can use Riemann Sums to evaluate definite integral  $\int_a^b f(x) dx$

Riemann's Sums use rectangle to approximate the area under curve.

Another useful integration rule is the Trapezoidal rule. Under this rule, the area under a curve is evaluated by



Pg (4)

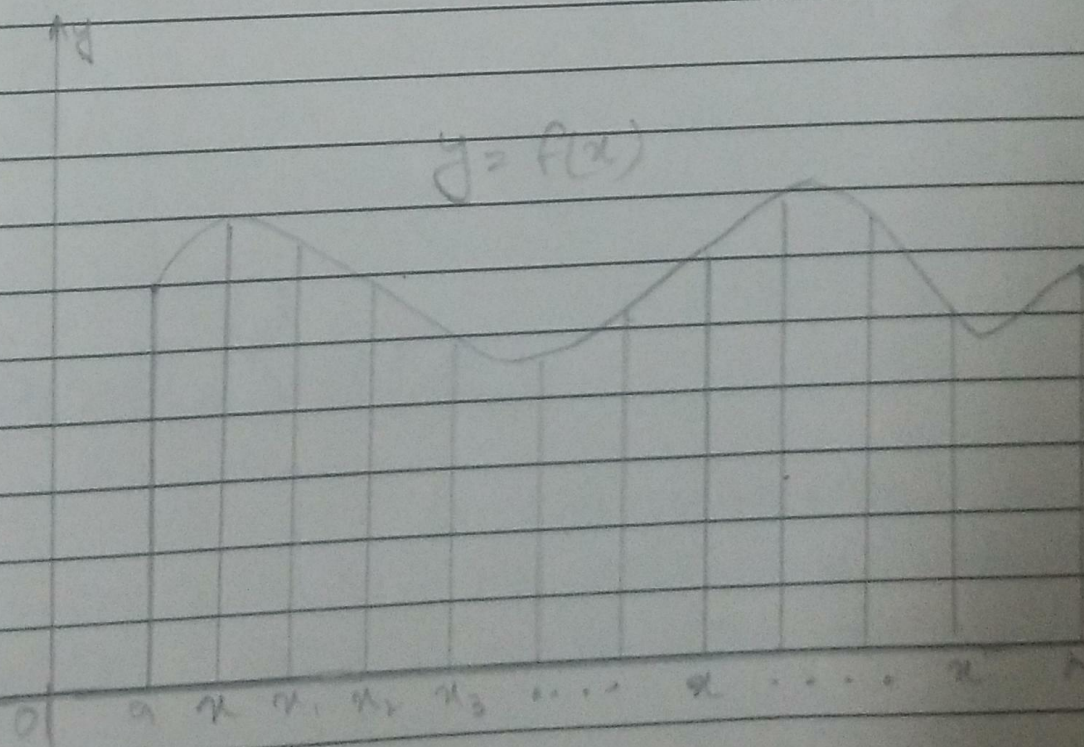
dividing the total area into little trapezoids rather than rectangles.

Let  $f(x)$  be continuous on  $[a, b]$ .  
we partition the interval  $[a, b]$   
into  $n$  equal sub intervals, each of  
width.

$$\Delta x = \frac{b-a}{n}$$

Such that

$$a = x_0 < x_1 < x_2 < \dots < x_n = b$$





The trapezoidal rule for approximating  $\int_a^b f(x) dx$  is given by

$$\int_a^b f(x) dx \approx T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)],$$

where

$$\Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x$$

As  $n \rightarrow \infty$  the right-hand side of the expression approached the definite integral

$$\int_a^b f(x) dx$$

## \* Simpson Rule :-

It is a numerical method that approximate the value of definite integral by using "quadratic function"



This method is named after the English mathematician Thomas Simpson (1710-1761)

Simpson's rule is based on the fact that given three points, we can find the equation of a quadratic through those points.

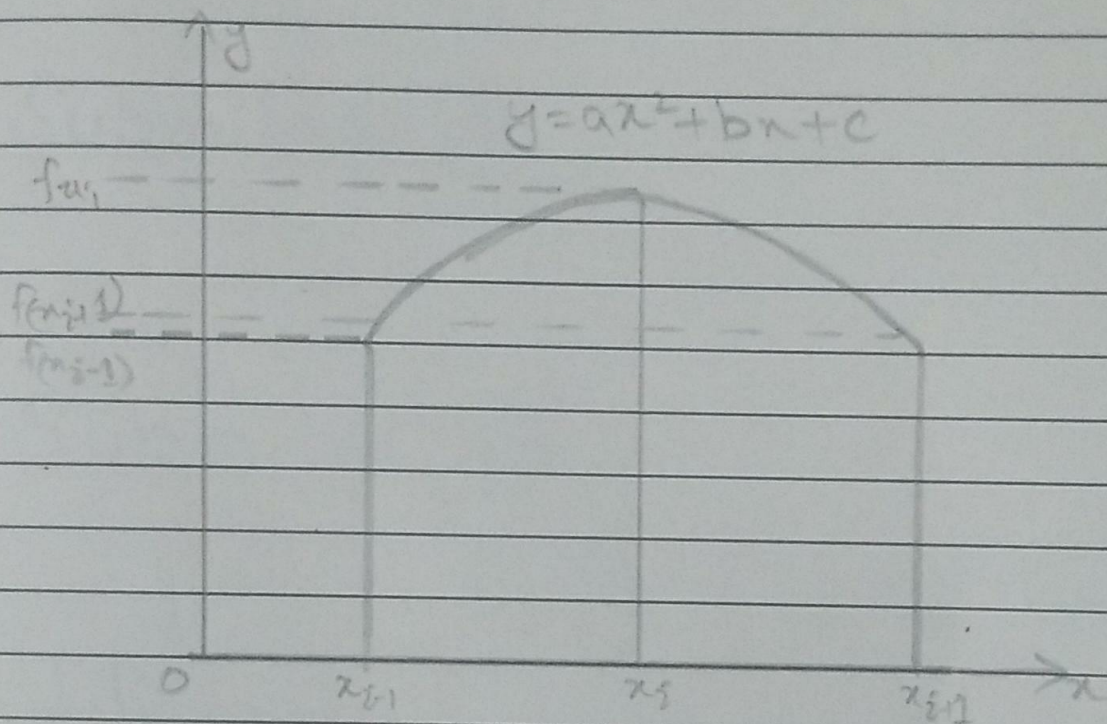
To obtain an approximation of the definite integral  $\int_a^b f(x) dx$  using Simpson's rule, we partition the interval  $[a, b]$  into an even number  $n$  of subintervals, each of width

$$\Delta x = \frac{b-a}{n}$$

On each pair of consecutive subintervals  $[x_{i-1}, x_i], [x_i, x_{i+1}]$ , we consider a quadratic function  $y = ax^2 + bx + c$  such that it passes through the points  $(x_{i-1}, f(x_{i-1})), (x_i, f(x_i)), (x_{i+1}, f(x_{i+1}))$



$P_9(7)$



If the function  $f(x)$  is continuous on  $[a, b]$ , then

$$\int_a^b f(x) dx \approx \Delta x/3 [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) \dots + 4f(x_{n-1}) + f(x_n)]$$

The coefficients in Simpson's rule have the following pattern.

$$\underbrace{1, 4, 2, 4, 2, \dots, 4, 2, 4, 1}_{n+1 \text{ points}}$$