Q#01:—

Sol:- The general form for SDOF System is

$$ku + c\dot{u} + m\ddot{u} = P(t).$$

In our System / Question is undamped ($c=0$) undergoing free vibration ($P(t)=0$).

Hence general EOM becomes $ku + m\ddot{u} = 0$ —①

$$K = 3EI/L^3$$

$$K = \frac{3 \times 2900 \ K/in^2 \times 150 \ in^4}{(10 \times 12 \ in)^3}$$

$$K = 7.55 \ K/in$$

In order to eliminate the chance of mistake during calculation, it is more appropriate to use fundamental units like lb, ft, sec or kg, m, sec

$$K = 7.55 \ K/in = 90625 \ lb/ft.$$

$$m = \frac{7712 \ lb}{32.2 \ ft/sec}$$

$$m = 239.50 \ slug.$$

Now $\omega_n = \sqrt{K/m}$

$$\omega_n = \sqrt{\frac{90625}{239.50}}$$

$\omega_n = 19.45$ rad/sec.

Also

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.45}$$

$T_n = 0.322$ sec.

Substituting the Corresponding values in eq ①

$$90625\, u + 239.50\, \ddot{u} = 0$$

where "k" is in lb/ft and "m" is in lb sec/ft^2

General Solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n \sin(\omega_n t)}$$

$u(0) = 1/2 = 1/24$ ft on $\dot{u}(0) = 0$.

$u(t) = (1/24) \times \cos(19.45t)$

Equivalent Static force at any time "t" is

$$f_s(t) = K \cdot u(t)$$

$$f_s(t) = \frac{90625 \times \cos(19.45 ft)}{24.}$$

$$f_s(t) = \cancel{\quad} \; 3776.041 \times \cos(19.45 ft).$$

Amplitude of dynamic displacement u_0 for undamped free vibration is

$$u_0 = \sqrt{\left((u(0))^2 + \left(\frac{\dot{u}(0)}{\omega n}\right)^2\right)}$$

$$u_0 = \sqrt{(1/24)^2 + 0}$$

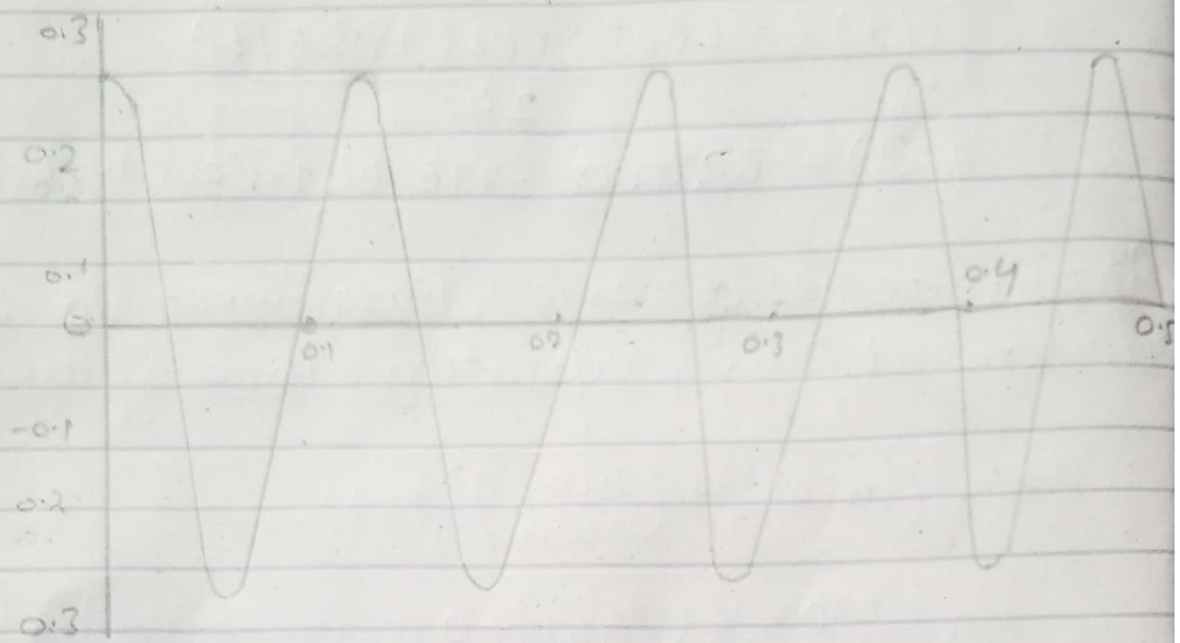
$$u_0 = 1/24 \text{ ft}$$

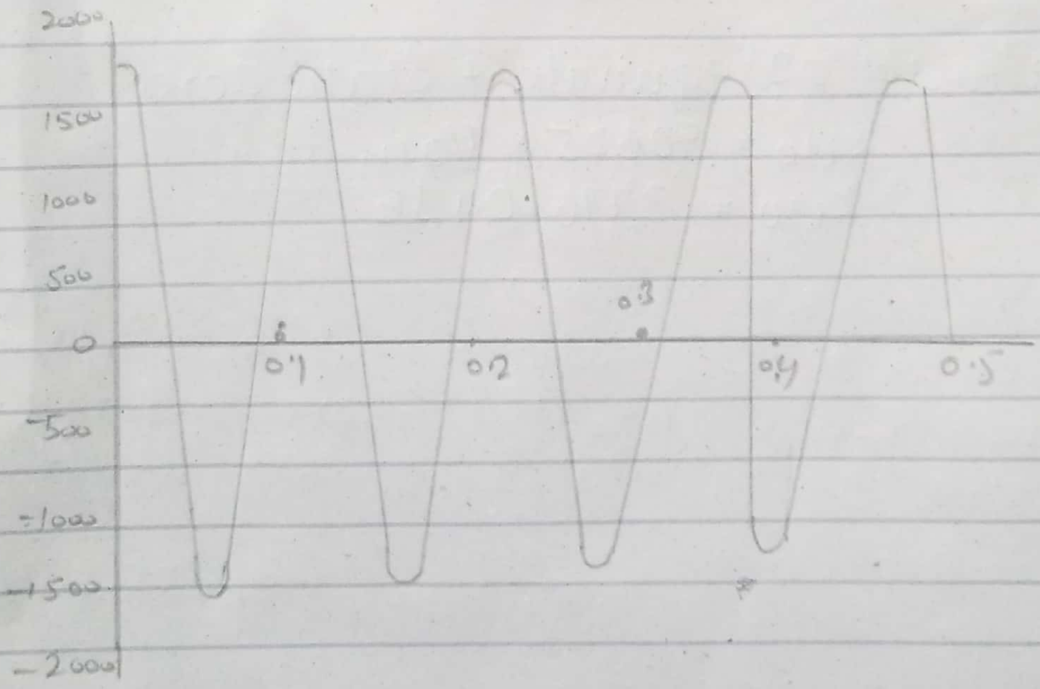
Amplitude of equivalent Static force.

$$K u_0 = 90625 \times 1/24$$

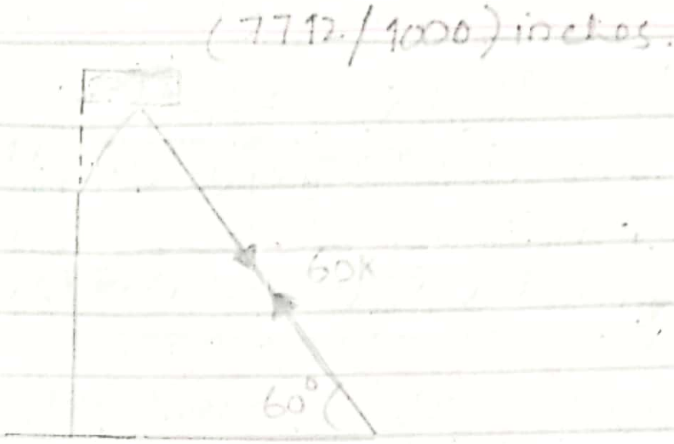
$$K u_0 = 3776.04 \text{ lb}$$

Scanned with CamScanner


0.3
0.2
0.1
0
-0.1
0.2
0.3
0.1
0.2
0.3
0.4
0.5


2000
1500
1000
500
0
-500
-1000
-1500
-2000
0.1
0.2
0.3
0.4
0.5

Q# 03:—



Sol:—

$$u_1 = \frac{7712}{1000} = 7.712 \text{ or } 7.7''$$

After $j = 7$, $u_{j+1} = u_6 = 2.286\,cm$
$= 0.9''$

a) ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$7 = \frac{1}{2\pi\zeta} \ln(7.7/0.9)$$

$$\boxed{\zeta = 0.049 = 4.9\%}$$

b) $T_n = ?$

7 cycles of vibrations are completed in 3.57 sec.

Time required to complete one cycle
$= 3.57/7 = T_D$

$T_D = 0.51$ sec

$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$2\pi/\omega_D = 2\pi/(\omega_n\sqrt{1-\zeta^2})$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.049)^2}$$

$$\Rightarrow \boxed{T_n = 0.5094 = 0.51 \, Sec}$$

c) $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{7.7} = 3.9 \, K/in$$

$$\boxed{K = 46800 \, lb/ft}$$

d) Weight of tank, $w = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(w/g)}} = \sqrt{\frac{K \times g}{w}}$$

$$\Rightarrow w = \frac{K \times g}{\omega_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$w = \frac{K g}{\left(\frac{4\pi^2}{T_n^2}\right)} = K g \times \frac{T_n^2}{4\pi^2}$$

$$w = \frac{46800 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$$w = 9928.5 \, lb$$

$$\boxed{w = 9.93 K}$$

e)- $c = ?$

It is known that $\zeta = \dfrac{c}{2m\omega_n}$

$$\Rightarrow c = \zeta \times 2m\omega_n = \zeta \times 2m \times (2\pi f_n)$$

$$c = 0.049 \times 2 \times 2 \times \left(\frac{\pi}{0.51}\right)\left(\frac{9928.5}{32.2}\right)$$

$$\boxed{c = 372.27 \ \text{lb.sec/ft}}$$

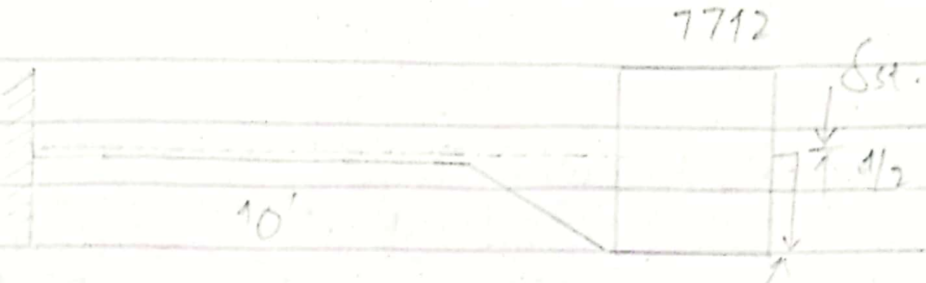
f)- No. of cycles to reduce displacement amplitude from 7.7in to 0.5", $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow j = \frac{1}{2 \times \pi \times 0.049} \ln\left(\frac{7.7}{0.5}\right)$$

$$\Rightarrow \boxed{j = 8.69 \text{ or } 9 \text{ cycles}}$$

Q#02:—



Sol:— E.O.M for damped free vibration is;

$K u + C \dot{u} + m \ddot{u} = 0$ —(1)

It is known from (question #01)

$K = 90625$ lb/ft and $m = 239.50$ slug

$\omega_n = 19.45$ $m = 239.50 \, lb \cdot sec^2/ft$

$\Rightarrow C = \xi \times 2 m \omega_n = 2 \times 239.50 \times 19.45 \times \xi$

($\xi = 0.03 - 0.05$
with considerable cracking
the damping ratio)

$\Rightarrow C = 2 \times 239.50 \times 19.45 \times 0.05$

$C = 465.82$ lb.sec/ft

By substituting values of K, C and m in eq (1)

$90625 u + 465.82 \dot{u} + 239.50 \ddot{u} = 0$.

Solution to the E.O.M for damped free vibration is;

$u(t) = e^{-\xi \omega_n t} \left(u(0) \cos(\omega_D t) + \frac{1}{\omega_D} \left(\dot{u}(0) + u(0) \cdot \xi \omega_n \right) \cdot \sin(\omega_D t) \right)$

$W_D = 19.45$ rad/sec.

$$u(t) = e^{-0.05\times 19.45}\left(\frac{1}{24}\times\cos(19.45t) + \frac{1}{19.45}\right.$$

$$u(t) = \left(0+\frac{1}{24}\times 0.05\times 19.45\times \sin(19.45t)\right)$$

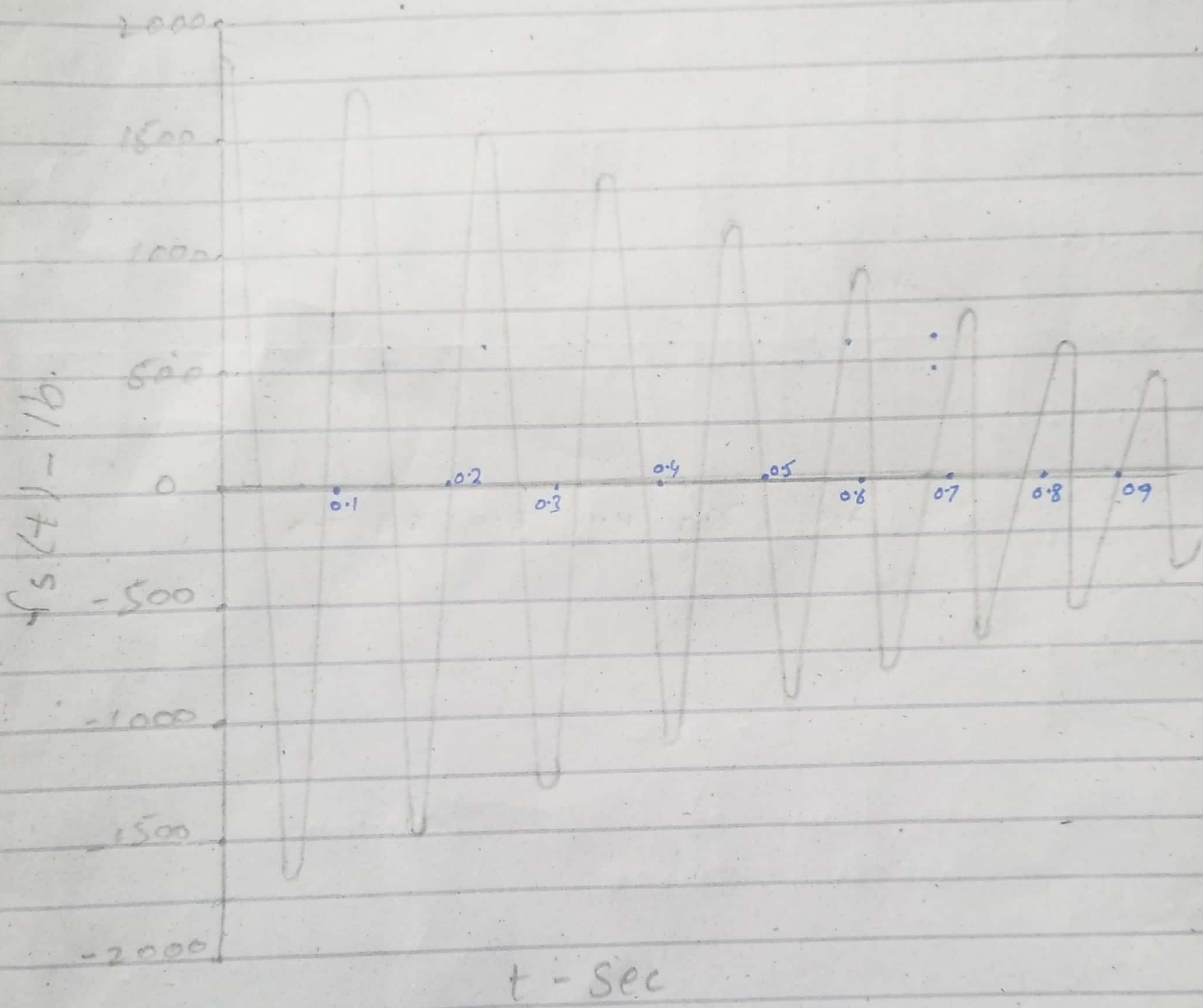
$$u(t) = e^{-0.972t}\left(0.0417\times\cos(19.45t) + 0.0514\times\sin(19.45t)\right)$$

$$u(t) = e^{-0.972t}\left(0.0417\times\cos(19.45t) + 0.021\times\sin(19.45t)\right.$$

$$f_s(t) = k\cdot u(t) = 90625\times u(t)$$

$$f_s(t) = e^{-1.373t}\left(3779\times\cos(19.45t) + 1903.12\times\sin(19.45t)\right.$$

Damped Free vibration:



Variation of equivalent static forces with time.