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Sec :- "C"

Dept :- BE(C)

Subject :- Applied calculas.

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Date :- 21-08-2020.

Q1:- The function $g(t)$ is defined by ①

$$g(t) = \begin{cases} 0 & t<0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3<t\le 4 \\ 12 & t>4 \end{cases}$$

a:- State any point of discontinuity.

b:- Find if they exist.

i:- $\lim\limits_{t-3} g$

Sol:-

a:- To check the possibility of the discontinuity of the function is at $t=0$ and 4.

→ First at $t=0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L.

$$\lim g(1+h) = \lim (1-h)^2$$

$$= \lim_{h\to 0} 1+h^2+2h$$

Apply limits.

$$= 1+0^2+2(0)$$

$$= \underline{1}$$

For L.H.L.

$$\lim_{h\to 0} g(1-h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3$$

$$= \lim_{h\to 0} 2-2h+3$$

Apply limit

$$= 2-2(0)+3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

Now at $t=4$

$$S(4) = 2(4)+3$$

$$= 8+3$$

$$= 11$$

For R.H.L.

$$\lim_{h\to 0} S(1+h) = \lim_{h\to 0} 2(1+h)+3$$

$$= \lim_{h\to 0} 2+2h+3$$

Apply limit

$$= 2+2(0)+3 = 5$$

For L.H.L.

$$\lim_{h\to 0} S(1-h) = 12$$

$g(4) = R.H.L \neq L.H.L$ ③

Point of discontinuity is at $t = 4$.

b) Find if they exist.

i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limit

$= 1 + 3^2 + 2(3) \Rightarrow \underline{1}6$

L.H.L. $\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

Apply limit.

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is $-$ive).

Q2:- Find the Maclaurin's series for: ①

$$Y(x) = x^2 + \sin x$$

Sol:- since we know that maclaurin series is,

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)(x - x_0)^2}{2!} + \cdots$$

Put $x_0 = 0$

$$y(x) = y(0) + (x-0)\,y'(0) + \frac{(x-0)^2 y''(0)}{2!} + \cdots$$

$$y(x) = y(0) + x\,y'(0) + \frac{x^2 y''(0)}{2!} + \cdots \rightarrow (1)$$

Now find

$y(0) = ?$

$y(x) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$= 0 + 0$

$$\boxed{y(0) = 0}$$

$y(x) = x^2 + \sin x$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$y'(x) = 2x + \cos x$

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$y'(0) = 2(0) + \cos(0)$

$= 0 + 1$

$\boxed{y'(0) = \underline{1}}$

Since $y'(x) = 2x + \cos x$

$\frac{d}{dx} y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$

$= 2 - \sin x$

$y''(x) = 2 - \sin x$

$y''(0) = 2 - \sin 0$

$= 2 - 0 = 2$

$\boxed{y''(0) = 2}$

Now

$y''(x) = 2 - \sin x$

$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$

$= 0 - \cos x$

$y'''(x) = 0 - \cos x$

$y'''(0) = -\cos(0)$

$\boxed{y'''(0) = -\underline{1}}$

Put in eq(i)

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \cdots$$

So

$$y(x) = x + x^2 - \frac{x^3}{3!} + \cdots.$$

Q3 (i):-

$$1+xy = x^2+y^2$$

Solution:-

$$1+xy = x^2+y^2$$

$$= \frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$= \frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$= 0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x^{2-1} + 2y^{2-1}\frac{dy}{dx}$$

$$= x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$= x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$= \frac{dy}{dx}(x-2y) = 2x-y$$

$$= \boxed{\frac{dy}{dx} = \frac{2x-y}{x-2y}} \longrightarrow y'$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{2x - xy - 4y + 2yy' - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$$

$$y'' = \frac{\cancel{2x} - xy' - 4y + \cancel{2yy'} - \cancel{2x} + 4xy' + y - \cancel{2yy'}}{(x-2y)^2}$$

$$\boxed{y'' = \frac{4xy' - 3y - xy'}{(x-2y)^2}}$$ Ans.

Q3 (2) :- Find y by using logarithmic differentiation.

$$Y = x^3 (1+x)^9 e^{6x}$$

Sol:-

$$\ln y = \ln \left(x^3 (1+x)^9 e^{6x}\right)$$

$$= \ln \left(x^3 (1+x)^9\right) + \ln e^{(6x)}$$

$$= \ln x^3 + \ln (1+x)^9 + 6x$$

$$= 3 \ln x + 9 \ln (1+x) + 6x$$

Now

$$\frac{d \ln (y)}{dx} = \frac{d}{dx} \left(3 \ln x + 9 \ln (1+x) + 6x\right)$$

$$= 3 \frac{d}{dx} \ln x + 9 \frac{d}{dx} \ln (1+x) + 6 \frac{dx}{dx}$$

$$= 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6$$

$$\boxed{\frac{d \ln (y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6}$$ Ans.