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(1)

Q1, Ans

SOL#

3rd ID = 4

$$x_1 - 4x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$\left[\begin{array}{ccc:c} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & +40 & -10 & 10 \end{array}\right] R_3 - 5R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right] \begin{array}{l} R_2/4 \\ R_3/10 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & -15 \end{array}\right] ; R_3 - 4R_2$$

Consistent of because of triangle (2)

$-15\ x_3 = -15$

$x_3 = 1$

$x_1 - 4x_3 = 4$

$x_2 = 4 + 4x_3$

$x_2 = 8$

$x_1 - \cancel{8}4\ x_2 + x_3 = 0$

$x_1 = 4x_2 - x_3$

$\boxed{x_1 = 60}$ Ans

(2)

Q2 Sol:

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & x=2.0 \\ 5 & -2 & 7 \end{bmatrix} \quad \boxed{4x\,2.0 = 8}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj(A) \Rightarrow [|A| \neq 0]$$

$$|A| = 3[(-1\times7)-(8\times(-2))] - 4[(2\times7)-(8\times5)] - +5\{(2\times(-2))-(-1\times5)\}$$

$$|A| = 3(-7-(-16)\} - 4\{(14-40)\} + 5\{-4-(-5)\}$$

$$|A| = 3[-7+16) - 4(-26) + 5(-4+5)$$

$$|A| = 3(9) - 4(-26) + 5(1)$$

$$|A| = 27 + 104 + 5$$

$$\boxed{|A| = 136}$$

Inverse of matrix $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$ Ex-3

Now $Adj(A) = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$

First to find Co factor

(3) (4)

Cofactor a_{13}

$\Rightarrow (-1)^{1+1} \times \text{minor}$

$\Rightarrow (-1)^2 \times \begin{bmatrix} -1 & 8 \\ -2 & 7 \end{bmatrix}$

$\Rightarrow (1) \times [(-1 \times 7) - (8 \times -2)]$

$\Rightarrow 1 \times (-7 + 16)$

$\Rightarrow 1\,(9)$

$\boxed{\Rightarrow 9}$

Cofactors a_{14} #

$\Rightarrow (-1)^{1+2} \times \text{minor}$

$= (-1)^3 \times \begin{bmatrix} 2 & 8 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow -1 \times [(2 \times 7) - (8 \times 5)]$

$\Rightarrow -1\,(14 - 40)$

$\Rightarrow -1\,(-26)$

$\boxed{= 26}$

Cofactor a_{15} #

$\Rightarrow (-1)^{1+3} \times \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

$\Rightarrow (-1)^4 \times [(2 \times -2) - (-1 \times 5)]$

$\Rightarrow 1 \times [(-4 + 5)]$

Cofactor of 2 #

5

$\Rightarrow (-1)^{2+3} \times \text{minor}$

$\Rightarrow (-1)^5 \times \begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix}$

$\Rightarrow -1 \{(3\times(-2)) - (4\times 5)\}$

$\Rightarrow -1 \{-6 - 20\}$

$\Rightarrow -1(-26)$

$\Rightarrow \boxed{26}$

Cofactor of 5 #

$\Rightarrow (1)^{3+1} \times \text{minor}$

$\Rightarrow (-1)^4 \times \begin{bmatrix} 4 & 5 \\ -1 & 8 \end{bmatrix}$

$\Rightarrow 1\times [(4\times 8) - (5\times(1))]$

$\Rightarrow 1\times \{32 + 5\}$

$\Rightarrow 1\times(37)$

$\boxed{=37}$

$\boxed{\text{Cofactor of } 2}$ #

$\Rightarrow (-1)^{3+2} \times \text{minor}$

$\Rightarrow (-1)^5 \times \begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow -1 \times \{(3\times 8) - (5\times 2)\}$

$\Rightarrow 1\ (1)$

$\boxed{= 1}$

(6)

(C)facters a_{12} #

$\Rightarrow (-1)^{2+1} \times \text{minor}$

$\Rightarrow (-1)^3 \times \begin{bmatrix} 4 & 5 \\ -2 & 7 \end{bmatrix}$

$\Rightarrow -1 \{(4\times7) - (-2\times5)\}$

$\Rightarrow -1 [28 + 10]$

$\Rightarrow -1 (38)$

$\boxed{\Rightarrow -38}$

(C)factors a_{21} #

$\Rightarrow [-1]^{2+2} \times \text{minor}$

$\Rightarrow (-1)^4 \times \begin{bmatrix} 3 & 5 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow (-1)^4 \times [(3\times7) - (5\times5)]$

$= 1 (21 - 25)$

$= 1 (-4)$

$\boxed{= -4}$

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$\Rightarrow -1\{24-10\}$

$\Rightarrow -1[14]$

$\boxed{\Rightarrow -14}$

Cofactors 7 7 #

$\Rightarrow (-1)^{3+3} \times \text{minor}$

$\Rightarrow (-6)^{6} \times \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}$

$\Rightarrow +1 \times [(3\times(-1)) - (4\times 2)]$

$= 1\{-3-8\}$

$= 1(-11)$

$\boxed{\Rightarrow 3\ -11}$

$$\text{Minors} = \begin{bmatrix} 9 & -26 & 1 \\ 38 & -4 & -26 \\ 37 & 14 & -11 \end{bmatrix}$$

&

$$\text{Cofactor} = \begin{bmatrix} 9 & 26 & 1 \\ -38 & -4 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

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Now we take the transpose of Co factors of matrix n

$$Adj(A) = \begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{(136)} \times \begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & 11 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 9/136 & -38/136 & 37/136 \\ 26/136 & -4/136 & -14/136 \\ 1/136 & 26/136 & -11/136 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 9/136 & -19/68 & 37/136 \\ 13/68 & -1/34 & -7/68 \\ 1/136 & 13/136 & -11/136 \end{bmatrix}$$

Ans

1) 3 Ans #

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$$2x + 2y + 4z = 18$$
$$x + 3y + 2x = 13$$
$$3x + 2y - 3z = 14$$

Converting given equation into matrix form.

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$R_1 \leftarrow R_1 \div 2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

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~~9~~ 10

$R_3 \leftarrow R_3 - 3 \times R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$R_2 \leftarrow R_2 + 2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$R_1 \leftarrow R_1 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$R_3 \leftarrow R_3 + R_2$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & -11 \end{array}\right]$$

$R_3 \leftarrow R_3 + -9$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

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$R_1 \leftarrow R_1 - 2 \times R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

$x = 4/9$

$y = 2$

$z = 11/9$ Answer

Q4 Ans #

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Find eigen values of the matrix A

$|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 2 & -2 \\ -5 & (3-\lambda) & 2 \\ -2 & 4 & (1-\lambda) \end{vmatrix} = 0$$

$\therefore (4-\lambda)((3-\lambda) \times (1-\lambda) - 2\times 4) - 2((-5)\times(1-\lambda) - 2\times(-2)) + (-2)(-$
$\times 4 - (3-\lambda) \times (-2)) = 0$

$= (4-\lambda)((3-4\lambda+\lambda^2)-8) - 2((-5+5\lambda) - (-4)) - 2(-20) - (-6+2\lambda$

$\therefore (4-\lambda)(-5-4\lambda+\lambda^2) - 2(-1+5\lambda) - 2(-14-2\lambda) = 0$

$\therefore (-20 - 11\lambda + 8\lambda^2 - \lambda^3) - (-2+10\lambda) - (-28-4\lambda) = 0$

$\therefore (-\lambda^3 + 8\lambda^2 - 17\lambda + 10) = 0$

$= -(\lambda-1)(\lambda-2)(\lambda-5) = 0$

$= (\lambda-1) = 0$ or $(\lambda-2) = 0$ or $(\lambda-5) = 0$

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1. Eigen vector for $\lambda = 1$

$$A - \lambda I = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - I \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix}$$

Now reduce the matrix interchange the row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -5 & 2 & 2 \\ 3 & 2 & -2 \\ -2 & 4 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 = -5$

$$\begin{bmatrix} 1 & -0.4 & -0.4 \\ 3 & 2 & -2 \\ -2 & 4 & 0 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 3 \times R_1$ (14)

$$= \begin{bmatrix} 1 & -0.4 & -0.4 \\ 0 & 3.2 & -0.8 \\ -2 & 4 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 2 \times R_1$

$$= \begin{bmatrix} 1 & -0.4 & -0.4 \\ 0 & 3.2 & -0.8 \\ 0 & 3.2 & -0.8 \end{bmatrix}$$

$R_2 \leftarrow R_2 \times 0.3125$

$$\begin{bmatrix} 1 & -0.4 & -0.4 \\ 0 & 1 & -0.25 \\ 0 & 3.2 & -0.8 \end{bmatrix}$$

$R_1 \leftarrow R_1 + 0.4 \times R_2$

$$\begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 3.2 & -0.8 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 3.2 \times R_2$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 0 & 0 \end{bmatrix}$$

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The System associated with eign value $\lambda = 1$

$$(\lambda - 1I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$= x_1 - 0.5x = 0, \; x_2 - 0.25x_3 = 0$

$= x_1 = 0.5x_3, \; x_2 = 0.25x_3$

$\therefore$ eigen vectors corresponding to the eign value $\lambda = 1$ is

$$V = \begin{bmatrix} 0.5x_3 \\ 0.25x_3 \\ x_3 \end{bmatrix}$$

let $x_3 = 1$

$$v_1 = \begin{bmatrix} 0.5 \\ 0.25 \\ 1 \end{bmatrix}$$

$\therefore$ Eigen vectors for $\lambda = 2$

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$$A - \lambda I = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix}$$

∴ Now reduce the matrix interchanging row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -5 & 1 & 2 \\ 2 & 2 & -2 \\ -2 & 4 & -1 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div -5$

$$= \begin{bmatrix} 1 & -0.2 & -0.4 \\ 2 & 2 & -2 \\ -2 & 4 & -1 \end{bmatrix}$$

$= x_1 - 0.5x_1 = 0 \cdot x_2 - 0.5x_3 = 0$

$= x_1 = 0.5x_3 \cdot x_2 = 0.5x_3$

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eigen values corresponding the eigenvalue of $\lambda = 2$ is

$$v = \begin{pmatrix} 0.5x_3 \\ 0.5x_3 \\ x_3 \end{pmatrix}$$

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

2. The eigen values of compose the column of matrix P

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1, The digonal of matrix D is composed of the eigenvalue

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

3, Now find P'

$$|P| = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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$R_2 \leftarrow R_2 = 0.2778$

$$\begin{bmatrix} 1 & -0.2 & -0.4 \\ 0 & 1 & -0.5 \\ 0 & 24 & -12 \end{bmatrix}$$

$R_1 \leftarrow R_1 + 0.2 \times R_2$

$$\therefore \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 24 & -12 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2.4 \times R_2$

$$= \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigen value $\lambda = 2$

$$(A - 2I) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{2} \times \begin{bmatrix} 1/2 & 1 \\ 1 & 1 \end{bmatrix} - \frac{1}{2} \times \begin{bmatrix} 1/4 & 1 \\ 1 & 1 \end{bmatrix} + 0 \times \begin{bmatrix} 1/4 & 1/2 \\ 1 & 1 \end{bmatrix}$$

$$= 1/2 = \left[(1/2 \times 1 - 1 \times 1) - \frac{1}{2} \times \left(\frac{1}{4} \times 1 - 1 \times 1\right) + 0 \times \left(1/4 \times 1 - \frac{1}{2} \times 1\right) \right.$$

$$= 1/2 \times (1/2 - 1) - \frac{1}{2} \times \left(\frac{1}{4} - 1\right) + 0 \times (1/4 - 1/2)$$

$$= 1/2 \left(-\frac{1}{2}\right) - \frac{1}{2} \times (-3/4) + 0 \times (-1/4)$$

$$= -1/4 + 3/8 - 0$$

$$= 1/8$$

$$Adj(D) = Adj \begin{bmatrix} 1/2 & \frac{1}{2} & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{bmatrix} 1/2 & 1 \\ 1 & 1 \end{bmatrix} & -\begin{bmatrix} 1/4 & 1 \\ 1 & 1 \end{bmatrix} & +\begin{bmatrix} 1/4 & 1/2 \\ 1 & 1 \end{bmatrix} \\ -\begin{bmatrix} 1/2 & 0 \\ 1 & 1 \end{bmatrix} & +\begin{bmatrix} 1/2 & 0 \\ 1 & 1 \end{bmatrix} & -\begin{bmatrix} 1/2 & 1/2 \\ 1 & 1 \end{bmatrix} \\ +\begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} & -\begin{bmatrix} 1/2 & 0 \\ 1/4 & 1 \end{bmatrix} & +\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} \end{bmatrix}$$

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$$= \begin{bmatrix} +(1/2 \times 1 - 1 \times 1) & -(1/4 \times 1 - 1 \times 1) & +(1/4 \times 1 - 1/2 \times 1) \\ -(1/2 \times 1 - 0 \times 1) & +(1/2 \times 1 - 0 \times 1) & -(1/2 \times 1 - 1/2 \times 1) \\ +(1/2 \times 1 - 0 \times \frac{1}{2}) & -(1/2 \times 1 - 0 \times 1/4) & +(1/2 \times \frac{1}{2} - 1/2 \times 1/4) \end{bmatrix}^T$$

$$= \begin{bmatrix} +(1/2 - 1) & -(1/4 - 1) & +(1/4 - 1/2) \\ -(1/2 + 0) & +(1/2 + 0) & -(1/2 - 1/2) \\ -(1/2 + 0) & -(1/2 + 0) & +(1/4 - 1/8) \end{bmatrix}^T$$

$$= \begin{bmatrix} -1/2 & 3/4 & -1/4 \\ -\frac{1}{2} & 1/2 & 0 \\ 1/2 & -1/2 & 1/8 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 3/4 & 1/2 & -1/2 \\ -1/4 & 0 & 1/8 \end{bmatrix}^T$$

Q5 Soln

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$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 2x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -4/3 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So we have a solution 7

$$x = \begin{bmatrix} 4/3\ 8 \\ 0 \\ 8 \end{bmatrix} = 8 \begin{bmatrix} \frac{4}{3} \\ 0 \\ 1 \end{bmatrix}$$

Q6 # SoL#

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$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Reduce matrix to reduced row-echilon form

Swap matrix row $R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in row R_2

Performing

$$R_2 \leftarrow R_2 - \frac{1}{3} R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in Row R3 by performing

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$$R_3 \leftarrow R_3 - \frac{1}{3} \cdot R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rank of a matrix is the number of ~~all~~ non all zero row

$$\text{Rank of} = \begin{bmatrix} 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix} = 2$$

Ans