

Final paper

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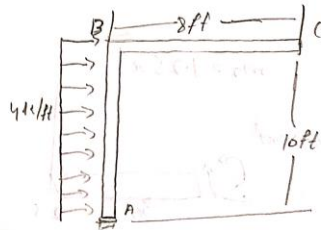
Section A

Date 27/6/20

Subject Structural Analysis-I

Q2

Determine the vertical displacement of free end point C on the frame shown in figure. Take $E = 29(10^3)$ KSI and $I = 600 \text{ in}^4$ for both members. Use method of virtual work.



Given Data:

uniform load = 4 k/ft

$E = 29 \times 10^3$ KSI

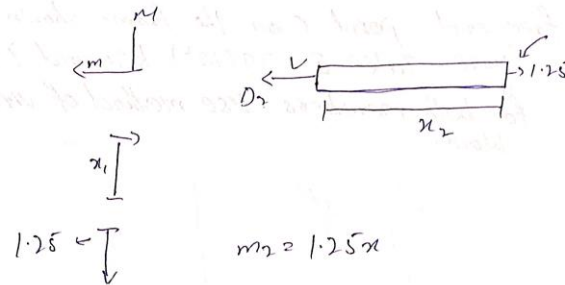
$I = 600 \text{ in}^4$

Required:

vertical displacement

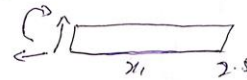
Solutions:

Now vertical moment

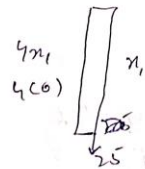


$$m_2 = 1.25x$$

Real moment



$$m_2 = 2.5x$$



$$m = \frac{40x_1 - \frac{1}{2}x_1(x_2)}{40x_1 - 2x_1^2}$$

Now by Virtual work equation

$$ADC = \int_0^L \frac{mMdx}{E}$$

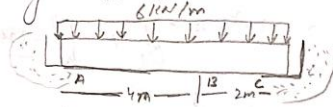
$$\Delta L = \int_0^{10} (1x_1) \frac{(40x_2 - 2x^2)}{E} dx + \int_0^{10} \frac{(1.25x^2)(2.5x^2)}{EI} dx$$

$$\Delta L = \frac{1}{EI} \left[\frac{42x^3}{3} - \frac{2x^3}{4} \right]_0^{10} + \left[\frac{(31.25x^4)}{8} \right]_0^{10}$$

$$\Delta L = 10649.6018$$

Q2

Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take $E = 200 \text{ GPa}$, $I = 60 \times 10^6 \text{ mm}^4$. Use Castigliano's Theorem.



Given

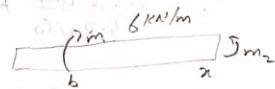
$$E = 200 \text{ GPa}$$

$$I = 60 \times 10^6 \text{ mm}^4$$

Required :

Slope and displacement

Solution



$$m_1 - m_2 = \frac{1}{2} (w_2) (6 + x_2)$$

$$m = m' + \frac{6x_2 + x_2^2}{2}$$

$$m = -m' + 3x_2 + \frac{x_2^2}{2}$$

taking partial derivation with respect to m

$$\frac{\partial m_2}{\partial P} = -x$$

$$\Delta B = \int_0^2 \frac{m(x)}{2P} \frac{dx}{E}$$

$$\Delta B = \frac{-3x^2(-x)}{EI} \Big|_0^2 + \int_0^1 \frac{-3x^2(-x) dx}{EI}$$

$$\Delta B = \frac{-3x^2}{4EI} \Big|_0^2 + \frac{-3x^4}{4EI} \Big|_0^1$$

put the value of EI and I

$$= \frac{-3x^2}{2(280)(60 \times 10^6)} \Big|_0^2 + \frac{-3x^4}{4(280)(60 \times 10^6)} \Big|_0^1$$

$$= \frac{-216 \text{ kN}\cdot\text{ft}^3}{4.8 \times 10^9} + \frac{-614.4 \text{ kN}\cdot\text{ft}^3}{4.8 \times 10^9}$$

$$= -4.5 \times 10^{-9} + (-1.28 \times 10^{-9})$$

$$\Delta B = 5.76 \times 10^{-10} \text{ inch}$$

Displacement

Slope ∴

$$m + \frac{1}{2} x (6x_1) = 0$$

$$m = -\frac{1}{2} x (6x_2) = 3x^2$$

So,

$$\frac{2m_1}{2m_1} = 0$$

$$m_1 - m_2 - \frac{1}{2} (x_2) (6 + x_2)$$

$$m_2 - m_2 - \frac{1}{2} x_2$$

$$m_2 = -m' + 6x_2 + x_2^2$$

$$m_2 = -m' + 3x^2 + x^2/2$$

$$\frac{2m_2}{2m_1} = -1$$

$$= \int_0^b \frac{-3x^2(4x)}{E \cdot I} + \int_0^b (-2 + 6x^2 + \frac{x^2}{2}) dx$$

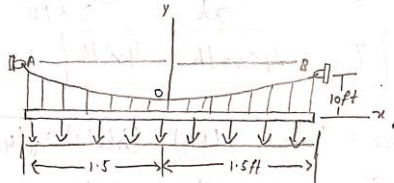
$$= 0 + \left(-x + \frac{6x^3}{3} + \frac{x^3}{6} \right) \Big|_0^b \left(\frac{1}{EI} \right)$$

$$= \frac{1}{200 \times (60 \times 10^6)} \left(-x + \frac{6x^3}{3} + \frac{x^3}{6} \right) \Big|_0^b$$

$$= \textcircled{1} = 4.125 \times 10^{-7} \text{ inch}$$

Q3

The cable is subjected to the uniform loading. If the slope of the cable at point O is zero, determine the equation of the curve and the force in the cable at O and B.



Given Data

$w =$ Uniform load $= 400 \text{ lb/ft}$

$h = 10 \text{ ft}$

$l = 15 \text{ ft}$

Required

Equation of curve and force in cable.

Solutions:

We know that

$$y = \frac{h}{l^2} x^2$$

$$y = \frac{h}{l^2} x^2$$

Putting the values

$$y = \frac{10}{(15)^2} x^2 = 0.44 x^2$$

$$T_0 = FH = \frac{W_0 L^2}{2h} = \frac{400(15)^2}{2 \times 10}$$

$$T_0 = 4500 \text{ lb} = 4.5 \text{ K}$$

$$T_{13} = T_{\max} = \sqrt{(FH)^2 + (hbd)^2} = \sqrt{(4500)^2 + (400 \times 15)^2}$$

$$T_{\max} = 7500 \text{ lb} = 7.5 \text{ K}$$

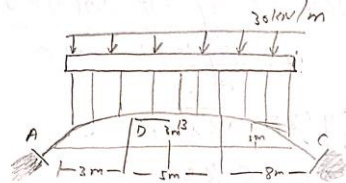
Now "T_{max}" by another equation

$$T_{13} = T_{\max} = W_0 L \sqrt{1 + \left(\frac{L}{2h}\right)^2} = 400 \times 15 \sqrt{1 + \left(\frac{15}{2 \times 10}\right)^2}$$

$$T_{\max} = 7500 \text{ lb} = 7.5 \text{ K}$$

Q4

The three-hinged spandrel arch is subjected to the uniform load of 30 kN/m . Determine the internal moment in the arch at point D.



Given Data:

uniform load = 30 kN/m

Required

Internal moment at D = ?

Solution

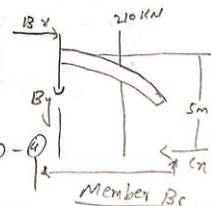
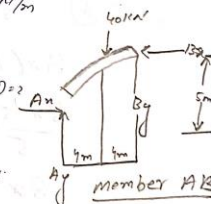
Dividing into two members:

AB and BC

AB

$$\sum \zeta M_i = 0$$

$$B_x(5) + B_y(8) - 240(4) = 0 \quad \text{--- (1)}$$



BC

$$\sum \text{M}_c = 0 \quad -B_x(0) + B_y(8) + 240(4) = 0 \rightarrow (6)$$

Adding eq (4) and (6)

$$B_x(5) + B_y(8) - 240(4) = 0$$

$$-B_x(5) + B_y(8) + 240(4) = 0$$

$$\hline 0 + 2B_y(8) + 0 = 0$$

$$2B_y(8) = 0$$

$$2B_y = 0 \text{ kN}$$

Putting the value of B_y in eq (6)

$$\text{eq (6)} \quad -B_x(5) + 0(8) + 960 = 0$$

$$B_x(4) = 960$$

$$\frac{B_x(5)}{5} = \frac{960}{5}$$

$$B_x = 192 \text{ kN}$$

Now at segment DB

$$\sum \text{M}_o = 0$$

$$192(2) - 186(2.5) - M_D = 0$$

$$384 - 375 - M_D = 0$$

$$9 - M_D = 0$$

$$\Rightarrow M_D = 9 \text{ kNm}$$