Name : Sayed Faiz Ali Shah

ID : 15247

Subject : Business Maths & Statistics

Semester : Final term (Summer)

Teacher : Sir Dr Liaqat Ali

Part 1

Q1.

i) (a) 1/8

ii) (b) 1/3

iii) (a) 1

iv) (a) $S = \Sigma y^2 - a\Sigma y - b\Sigma xy$

v) (b) $aE(x) + b$

vi) (d) None of the above

vii) (c) 4

viii) (a) -10

ix) (c) 3/4

x) (a) $P(A) + P(B)$

Q2:-

Answer:- $f(x) = \frac{3}{4}(3-x)(x-5),\ 3 \le x \le 5$

Find the variance, S.D

Sol:-

$$f(x) = \frac{3}{4}(3-x)(x-5) \quad 3 \le x \le 5$$

$$= \frac{3}{4}(3x - 15 - x^2 + 5x)$$

$$= \frac{3}{4}(-x^2 + 8x - 15)$$

$$\text{variance} = E(x^2) - \{E(x)\}^2$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= -\frac{3}{4}\int_3^5 x(-x^2 + 8x - 15)\,dx$$

$$= -\frac{3}{4}\int_3^5 x^3\,dx + \frac{3}{4}\int_3^5 8x^2 - \frac{3}{4}15\int_3^5 x\,dx$$

$$= -\frac{3}{4}\cdot\frac{x^4}{4}\Big|_3^5 + \frac{3}{4}\times 8\ \frac{x^3}{3}\Big|_3^5 - \frac{3}{4}\times 15\ \frac{x^2}{2}\Big|_3^5$$

$$= -\frac{3}{16}\cdot x^4\Big|_3^5 + 2\cdot x^3\Big|_3^5 - \frac{45}{8}\ x^2\Big|_3^5$$

$$= -\frac{3}{16}\{(5)^4 - (3)^4\} + 2\{(5)^3 - (3)^3\} - \frac{45}{8}\{(5)^2 - (3)^2\}$$

$$= -\frac{3}{16}(625 - 81) + 2(125 - 27) - \frac{45}{8}(25 - 9)$$

$$= -\frac{3}{16}(544) + 2(98) - \frac{45}{8}(16)$$

$$= -3(34) + 2(98) - 45 \times 2$$

$= -102 + 196 - 90$

$= E(x) = 4$

$= E(x^2) = \int_{\infty}^{\infty} x^2 f(x)\, dx$

$= \frac{3}{4} \int_{3}^{5} (x^2(-x^2+8x-15)\, dx$

$= \frac{3}{4} \int_{3}^{5} (-x^4+8x^3-15x^2)\, dx$

$= -\frac{3}{4} \int_{3}^{5} x^4\, dx + \frac{3}{4} + 8 \int_{3}^{5} x^3\, dx - \frac{3}{4} + 15 \int_{3}^{5} x^2\, dx$

$= \frac{-3}{4} \frac{x^5}{5}\Big|_3^5 + 6 \frac{x^4}{4}\Big|_3^5 - \frac{45}{4} \frac{x^3}{3}\Big|_3^5$

$= \frac{-3}{20}\{(5)^5-(3)^5\} + \frac{6}{4}\{(5)^4-(3)^4\} - \frac{45}{12}\{(5)^3+(3)^3\}$

$= -\frac{3}{20}(3125-243) + \frac{6}{4}(625-81) - \frac{45}{12}(125-27)$

$= -\frac{3}{20}(2882) + \frac{6}{4}(544) - \frac{45}{12}(98)$

$= -\frac{4323}{10} + 6 \times (136) - \frac{45 \times (49)}{6}$

$= -\frac{4323}{10} + \frac{816}{1} - \frac{2205}{6}$

$= -\frac{12969 + 24480 - 11025}{30}$

$= \frac{486}{30}$

$\Sigma x^2 = \frac{81}{5}$

(3)

$$\text{Var}(x) = Ex^2 - \{E(x)\}^2$$

$$= \frac{81}{5} - (4)^2$$

$$= \frac{81}{5} - 16$$

$$= \frac{81 - 80}{5}$$

$$= \frac{1}{5}$$

$$= 0.2$$

$$S.D = \sqrt{\text{Var}(x)}$$

$$= \sqrt{1/5}$$

$$\Rightarrow \sqrt{0.2}$$

$$= 0.44 \quad \text{Ans}$$

Q3-(a)

U = Set of odd numbers less then 25

A = Set of numbers divisible by 3 less then 19.

B = Set of numbers divisible by 5 or 10 less then 15.

C = The set of numbers which are multiples of 3 and less then 21.

Then find the following

(a) Show that $(A \cup B)^c = (A^c \cap B^c)$

If $U = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23\}$

$A = \{3, 6, 19, 12, 15, 18\}$

$B = \{10\}$

$C = \{3, 6, 9, 12, 15, 18\}$

Then

$(A \cup B)^c = A^c \cap B^c$

Proving that

L.H.S =>

$(A \cup B)^c = U / (A \cup B)$

$= \{1, 3, 5, 7 \ldots\ldots 23\} / \{3, 6, 9, 12, 15, 18\}$

$= U \{1, 3, 5, 7, \ldots\ldots 23\} / \{3, 6, 9, 10, 12, 15, 18\}$

$(A \cup B)^c = \{1, 5, 11, 13, 17, 19, 21, 23\}$ — (i)

R.H.S

$A^c = U/A$

$= \{1, 3, 5, \ldots 23\} / \{3, 6, 9, 12, 15, 18\}$

$= \{1, 5, 7, 11, 13, 17, 19, 21, 23\}$

$B^c = U/B$

$= \{1, 3, 5 \ldots 23\} / \{10\}$

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23\}$

$A^c \cap B^c = \{1, 5, 11, 13, 19, 21, 23\}$ — (ii)

So from eq (i) and eq (ii) we get that

L.H.S = R.H.S

$(A \cup B)^c = A^c \cap B^c$

H. Proved.

Q3 – (b)

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

We have to Prove that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S ⇒

$A \cap (B \cup C) = \{3,6,9,12,15,18\} \cap [\{10\} \cup \{3,6,9,12,15,18\}]$

$= \{3,6,9,12,15,18\} \cap \{3,6,9,10,12,15,18\}$

$= \{3,6,9,12,15,18\}$ — i

R.H.S ⇒

$(A \cap B) \cup (A \cap C) =$

$= [\{3,6,9,12,15,18\} \cap \{10\}] \cup [\{3,6,12,15,18\} \cap \{3,6,9 \ldots 18\}]$

$= \{3,6,9,12,15,18\}$ — ii

From eq (i) and eq (ii)

L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proved.

Q4:-

Solution:-

x	y	xy	x^2
5	16	80	25
6	19	114	36
8	23	184	64
10	28	280	100
12	36	432	144
13	41	533	169
15	44	660	225
16	45	720	256
17	50	850	289
102	302	3853	1308

$\hat{y} = a + bx$

$a = \bar{y} - b\bar{x}$

$b = \dfrac{\Sigma xy - \dfrac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}$

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{102}{9} = 11.33$

$b = \dfrac{3853 - \dfrac{(102)(302)}{9}}{1308 - \dfrac{(102)^2}{9}}$

$b = \dfrac{3853 - 3422.6}{1308 - 1156} = \dfrac{430.4}{152} = 2.83$

$a = \bar{y} - b\bar{x}$

$\bar{y} = \frac{\Sigma y}{n}$

$= \frac{302}{9}$

$= 33.5$

$\bar{x} = 11.33$

$b = 2.83$

$a = \bar{y} - b\bar{x}$

$a = 33.5 - 11.33(2.83)$

$a = 33.5 - 32.064$

$a = 1.4361$

$\hat{y} = 1.4361 + 2.83X$

This indicates that the value of Y increase by 2.83/unit for one unit increase in X.

Q5 :-

Solution :-

	x	Log x
1	9	0.9542
2	12	1.0792
3	15	1.1761
4	15	1.1761
5	16	1.2041
6	18	1.2553
7	20	1.3010
8	20	1.3010
9	25	1.3979
10	30	1.4771

$$G.M = \text{Antilog}\left(\frac{\Sigma \log x}{n}\right)$$

$$= \text{Antilog}\left(\frac{12.331}{10}\right)$$

$$= \text{Antilog}(1.2322)$$

$$= 17.06$$

Medium = $\left(\frac{n+1}{2}\right)^{th}$ item

$\left(\frac{11}{2}\right)^{th}$ item

$= 5.5^{th}$ item

$= \frac{5^{th} \text{ item} + 6^{th} \text{ item}}{2}$

$= \frac{16 + 18}{2}$

$= \frac{34}{2}$

Median = 17

Mode = 20 and 15

(Most repeated value)