

Name

Mansoor Rashid

ID

7698

Date

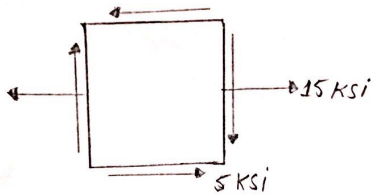
26/9/2020

Subject

Mechanics of Solid-II

Submitted to

Engr Usama Ali

Given data

$$\sigma_x = 15 \text{ ksi}$$

$$\sigma_y = 0$$

$$\tau_{xy} = -5 \text{ ksi}$$

Required Data.

- principal stress
- Max-plan Shear stress
- average Normal stress

Solution..

(a) principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{15+0}{2} \pm \sqrt{\left(\frac{15-0}{2}\right)^2 + (-5)^2}$$

$$\sigma_{1,2} = 7.5 \pm 9.01$$

$$\sigma_1 = 16.51 \text{ ksi}$$

$$\sigma_2 = 7.5 - 9.01$$

$$\sigma_2 = -1.51 \text{ ksi}$$

Now we find orientation we know that

$$2\alpha_2 = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$2\alpha_2 = \frac{-5}{(15-0)/2}$$

$$\alpha_2 = 0.33$$

Now we check which angle goes with which principle stress

We know that

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$$

$$= \frac{15+0}{2} + \frac{15+0}{2} \cos 2(0.33) + (-5)$$

$$\sin 2(0.33)$$

$$= \frac{15}{2} + \frac{15}{2} (0.99) + (-5) (0.12)$$

$$= 14.925 + 0.6$$

$$\sigma_{x_1} = 15.525$$

(b) Max-plan shear stress:

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{15 - 0}{2}\right)^2 + (-5)^2}$$

$$\tau_{max} = 9.01 \text{ KSI}$$

Now we find orientation we know that

$$\tan 2\phi = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\phi = +1.5$$

$$\phi = \tan^{-1}(+1.5)$$

$$2\phi = 56$$

$$\phi = \frac{56}{2}$$

$$\boxed{\phi = 28}$$

we know that

$$\tau_{x'y'} = \frac{-\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi$$

$$= \frac{15-0}{2} \sin 2(28) + (-5) \cos 2(28)$$

$$= -7.5 (0.82) - 28$$

$$= -8.95$$

Q1) part b

$$\sigma_x = 15 \text{ ksi}$$

$$\tau_{xy} = -5 \text{ ksi}$$

$$\sigma_y = 0$$

$$h = \frac{\sigma_x + \sigma_y}{2} = \frac{15-0}{2}$$

$$h = 7.5 \text{ ksi}$$

Radius

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{15-0}{2}\right)^2 + (-5)^2}$$

$$R = 9.01$$

Scale

$$1 \text{ small box} = 0.5 \text{ ksi}$$

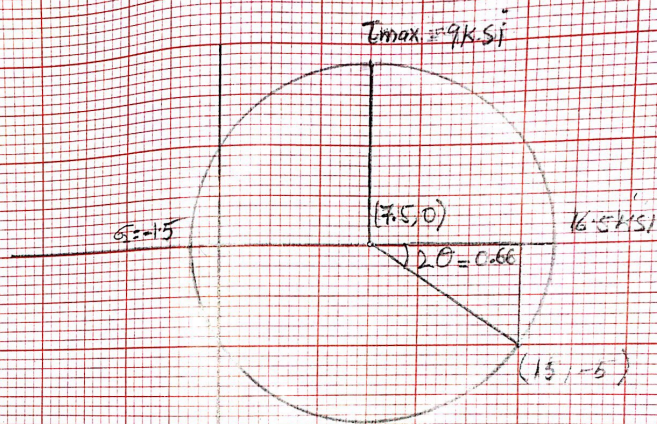
$$3 \text{ ksi} = 1 \text{ cm}$$

$$h = 7.5 \text{ ksi} = 2.5 \text{ cm}$$

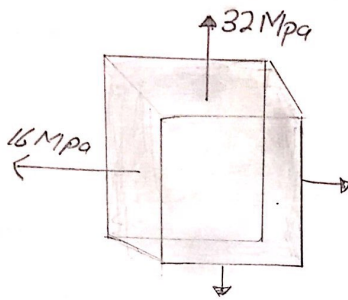
$$2x = 1.5 \text{ ksi} = 5 \text{ cm}$$

~~$$2x = 1.5 \text{ ksi} = 5 \text{ cm}$$~~

Scale = 1 small box = 0.5 ksi



Q No 2



Given data

$$\sigma_1 = 32 \text{ Mpa}$$

$$\sigma_2 = 16 \text{ Mpa}$$

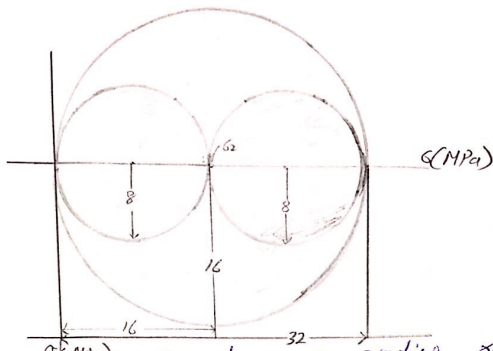
Required

Maximum shear stresses at point using Mohr's circle = ?

Solution

An orientation of an element 45° within this plane yields the state of absolute maximum shear stresses and the associated average normal stress namely:

- These stresses are plotted along the axis
- three Mohr's circle are constructed, stresses are shown.



The largest circle has a radius of 16 MPa and describes the state of stress in the plane only containing $\sigma_1 = 32 \text{ MPa}$

Absolute Max Shear stress and associated avg normal stress are

$$\tau_{\text{abs max}} = 16 \text{ MPa}$$

$$\sigma_{\text{avg}} = 16 \text{ MPa}$$

$\tau_{\text{abs max}}$ can be obtained from the equation

$$\tau_{\text{abs max}} = \frac{\sigma_1}{2} = \frac{32}{2} = 16 \text{ MPa}$$

$$\sigma_{\text{avg}} = \frac{32 + 0}{2} = 16 \text{ MPa}$$

By comparison the maximum in-plan shear ~~plan~~ stress can be determined from the Mohr's circle

drawn b/w $\sigma_1 = 32 \text{ Mpa}$ and $\sigma_2 = 16 \text{ Mpa}$

This gives a value of

$$\tau_{\text{inplan}}^{\text{max}} = \frac{32-16}{2} = 8 \text{ Mpa}$$

$$\sigma_{\text{avg}} = \frac{32+16}{2} = 24 \text{ Mpa}$$

Ans

Main Stresses responsible for failure of ductile and brittle materials

Stress responsible for failure of ductile and brittle materials are shear stress and tensile stress.

→ Ductile materials are limited by their shear strength. Ductile material usually fails because the shear stress exceeds the strength of ductile materials.

→ Brittle materials are limited by their tensile strength. Brittle materials fail when the tensile stress exceeds the strength of materials.

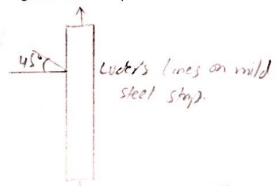
Two failure theories for Ductile Material:① Maximum shear-stress Theory

According to this Theory failure in ductile material occurs when the maximum shear stress in the part exceeds in shear stress in a part exceeds the shear stress in tensile test specimen (of same material at yield).

The slipping that occurs is caused by shear stress.

The edge of planes slipping as they appear on the surface of the strip are referred as Lüder's Line. These lines clearly indicate the slip plane in the strip which occur at approximately 45° with axis of the strip. The maximum sheare stress can be determined by drawing Mohis circle for the element. The result indicate that

$$\tau_{\max} = \frac{\sigma Y}{2}$$



This theory can be used to predict the failure stress of ductile material subjected to any type of loading.

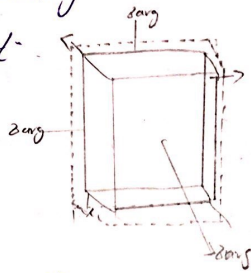
② Maximum Distortion Energy Theory

According to this theory "failure occurs when ~~the~~ an external loading will deform a material causing it to store energy internally throughout its volume. The energy per unit volume of material ~~is~~ ~~subjected~~ ~~to~~ ~~a~~ is called strain density energy.

$$U = \frac{1}{2} \sigma \epsilon$$

The strain energy density can be considered as the sum of two parts, one part representing the

The energy needed to cause a volume of energy of the element with no change in shape and the other part representing the energy needed to distort the element.



Theories of Failure for Brittle M.

① Maximum-Normal Stress Theory:

According to this theory "A brittle material will fail when the maximum tensile stress σ_1 in the material reaches a value that is equal to ultimate normal stress the material can sustain when it is subject to simple tension."

$$|\sigma_1| = \sigma_{ult}$$

$$|\sigma_2| = \sigma_{ult}$$



② Mohr's failure Criterion.

In some brittle materials tension and compression property are different when this occurs a criterion based on use of Mohr's circle may be used to predict failure.

This method was developed by Otto Mohr and is sometimes referred to as Mohr's failure criterion.

A uniaxial tensile test and uniaxial compressive test are used to determine the ultimate tensile and compressive stresses (σ_{ult} and material ultimate shear stress τ_{ult}). Mohr's circle for each of these stress conditions is then plotted in a diagram.

→ Which failure is more applicable on following material why?

(a) Steel.

(b) Concrete.

Concrete Maximum normal stress theory is applicable on concrete because tensile stresses are considered and as concrete is strong in compression and weak in tension. Also concrete is a brittle material.

→ Mohr's failure criterion theory is applicable to predict

the failure of brittle material as concrete is brittle material.

Steel steel is a ductile material and due to maximum shear stress steel bends which may cause the breaking of steel therefore maximum shear stress theory and maximum ϕ . distortion Theory are applicable to ductile material such as steel.