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Section : B

Department : Be (C)

Subject : Diffrential Equations

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(i) $m \times m$

(ii) One

(iii) $\begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} = a \times 1 - 2 \times 4 = a - 8 = 0$

$\Rightarrow a = 8$

(iv) $|A| = 2i(-i) - i(i)$

$= -2i^2 - i^2$

$= -2(-1) - (-1)$

$= 2 + 1$

$= 3$

(v) Scalar Matrix

(vi) $\frac{dy}{dx} + 2xy = y$

Separating variables

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1-2x)$

$\frac{dy}{y} = dx(1-2x)$

$\int \frac{1}{y} dy = \int 1 dx - \int 2x dx$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$\Rightarrow \ln y = x - x^2 + c$$

$$\Rightarrow \boxed{\ln y = x - x^2 + c}$$

vii

Order = 1

Degree = 3

viii

Order = 2

Degree = 1

ix

x

$$1\begin{bmatrix} b & b^2 \\ c & c^2 \end{bmatrix} - a\begin{bmatrix} 1 & b^2 \\ 1 & c^2 \end{bmatrix} + a^2\begin{bmatrix} 1 & b \\ 1 & c \end{bmatrix}$$

$$bc^2 - b^2c - a(c^2 - b^2) + a^2(c-b)$$

$$bc(c-b) - a(c-b)(c+b) + a^2(c-b)$$

$$(c-b)(bc - a(c+b) + a^2)$$

$$(c-b)(bc - ac - ab + a^2)$$

Q2 :- Part A :-

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the Product of factor which are linear in a, b, c

Sol :-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

(4)

$ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^2cb^3 - a^3b^2c$

Take common abc

$abc\,(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$

$\Rightarrow \boxed{abc\left[bc(c-b) - ac(c-a) + ab(b-a)\right]}$

Ans

Q2:- Part B:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic eq $\rightarrow |A - \lambda I| = 0 \rightarrow ①$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Expand by R_1

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow B$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$3-\lambda\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix}-(-1)\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix}-1\begin{vmatrix}-1 & 3-\lambda\\ -1 & -1\end{vmatrix}$$

$$(3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big)+1\big((-1)(2-\lambda)-(-1)(-1)\big)$$
$$-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$=(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(+1+3-\lambda)$$

$$=(3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$=3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$=\boxed{-\lambda^3+8\lambda^2-18\lambda+8}\rightarrow \text{(a)}$$

$$\Rightarrow \quad +1\begin{vmatrix}-1 & -1 & -1\\ -1 & 3-\lambda & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}$$

Expand by C_1

(8)

$$-1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow \quad -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-1\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -1\left(-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right)$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

Put a, b & c in B

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

by synthatic division

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \quad \Rightarrow \quad \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)=0$$

$$\lambda=4, \lambda=4$$

$$\boxed{\lambda_1=0, \lambda_2=2, \lambda_3=4, \lambda_4=4}$$ Ans

Q3:
$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$
$$x = 2\ ,\ y = 6$$

Sol:
$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$
$$(x^2+3y^2)\,dx = 2xy\,dy$$

Divide both sides by $2xy\,dx$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow ☆$$

let $y = vx$

Diff:
$$dy = v\,dx + x\,dv$$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \rightarrow (3)$$

Put (3) in (*)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{x}{xv} + 3\,\frac{vx}{x}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiply both sides by "2"

$$2v + 2x\,\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2v\,\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\,\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\,\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiply both sides by $\frac{dx}{dv}$

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiply both sides by $\frac{v}{x(1+v^2)}$

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

Put $v = y/x$

$$1+\left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \rightarrow (\star\star)$$

Put $x=2$, $y=6$ in eq $(\star\star)$

So

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Take "$\sqrt{\ }$" on both sides

$$\boxed{y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}}$$ Ans

or

$$\boxed{y = \pm x\sqrt{5x-1}}$$

Ans