Linear Algebra
BS-SE 2nd Semester

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Pray for the safety of humanity.
Pray for all the Muslims and Pakistanis wherever they are. Pray for our university, staff and students.

Instructions:

This is an open-book take-home mid-term assignment, to be submitted till Monday, April 26th, 2020. You may consult the textbook, your notes, and any material posted on SIC. No other sources of information are allowed, including friends, classmates, materials from other classes, tutors, etc. Please write your solutions as clearly and neatly as possible. Also, show all your work, preferably with explanations for each step. If you are asked to do a problem a specific way (for example, "use the standard matrix representation. . . "), then you will receive no credit for doing it any other way. You will also receive no credit for answers without sufficient work to produce them. Attempt all questions. Answers copied will both be marked zero. Late submission will not be accepted and marked zero.

How to Submit?

1. ***Write your names and Ids at the top of answer sheet.***
2. ***Scan / Take Photo of each paper and save each photo with a number. E.g. photo of paper 1 of answer sheet be saved with name 1.jpg, then 2.jpg and so on.***
3. ***Put all answer photos in a word file by simply copy and pasting images, name the document with subject name, your name and id e.g. LA_Ali_12345.***
4. ***You will be provided upload link on sic to submit your answers.Go to Lectures section and click on Upload Assignment and upload your answers document file in the subject.Different formats are mentioned for uploading assignment, student can choose any one of them.***

5.Due date and remaining time will be shown on the same window.

Q. No. 1 Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve this system. Where ID3 is the 3rd digit in your ID and ID_last is the last digit of your ID in inverse e.g. if your ID is 12345 then –ID_last = -5.

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\_Last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Q. No. 2

(a) Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

(b) Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail.

a. $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \Pi & 0 & 0 \\ 0 & 0 & -\Pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

b. $\begin{bmatrix} 1 & 0 & \Pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

c. $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

d. $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Q. No. 3

(a) The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.

(b) Find an echelon form for the below matrix using row operations. Where ID2 is 2nd digit in your ID e.g. if your ID is 12345 ID2 = 2, ID3=3, ID_first_last is the first and last digit of your ID i.e.15

$$\begin{bmatrix} 1 & \mathrm{ID2} & 8 \\ 2 & 8 & -1 \\ -\mathrm{ID3} & 0 & 0 \\ 1 & -4 & \mathrm{ID\_First\_Last} \end{bmatrix}$$

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SUBJECT:- LINEAR ALGEBRA

TEACHER SIR SHAKEEL

DEPARTMENT BE(E): Pg:01

Question No 1:-

Consider the given below matrix the augmented matrix of a linear system. Explain in your words the next elemantory row operation that should be performed in order to solve this system. Where ID3 is the 3rd digit of your ID and ID_last is the last digit of your ID in inverse if your ID is 12345 ID-last = -5.

~~SOLUTION:-~~

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\_last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

SOLUTION:-

My ID 5534

$$\left[\begin{array}{cccc|c} 1 & 3 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$\begin{cases} x_1 + 3 \times x_2 + 3 \times x_3 = 5 \\ x_2 - 4 \times x_3 = 7 \\ x_3 = -6 \quad (1) \\ x_4 = 3 \end{cases}$$

Find the variable x_4 from the eq 4 of the system (1).

$x_4 = 3$

Find the variable x_3 from the eq 3 of the system (1):

$x_3 = (-6)$

Find the variable x_2 from eq 2 of the system (1)

$x_2 = 7 + 4 \times x_3$

$= 7 + 4 \times (-6)$

$\boxed{= -17}$

Find the variable x_1 from eq 1 of the system (1).

$x_1 = 5 - 3 \times x_2 - 3 \times x_3$

$= 5 - 3 \times (-17) - 3 \times (-6)$

$= 74$

ANSWER:-

$x_1 = 74$

$x_2 = -17$

$x_3 = -6$

$x_4 = 3$

GENERAL SOLUTION:-

$$X = \begin{bmatrix} 74 \\ -17 \\ -6 \\ 3 \end{bmatrix}$$

Question No:-2

PART(A) Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

SOLUTION:-

Let first Matrix be "A"

Let second ~~Matrix~~ Matrix be "B"

ELEMENTARY ROW OPERATION

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$\underline{R_3 \sim R_3 - 2R_2}$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

REVERSE ROW OPERATION:-

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\underline{R_3 \sim 2R_2 + R_3}$$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

Hence Proved

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Part (B) Given below are some matrices. Find whether these are ~~used~~ in forms written infront of them or not. Explain in your own words for each of the selection in detail.

a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Part (A)

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$ is in echelon form:-

SOLUTION:-

As $A = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$

Yes, matrix A is in echelon form because of its definition. ~~Ete~~ Echelon matric states that. If a coloumn contains a leading entry than all entries below that leading entry are "zero"

So matrix A satisfies the echelon form of a matrix, So it is in echelon form.

PART (B)

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ is in echelon form:-

SOLUTION:-

As $B = \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Yes, Matrix B is in echelon form because of its definition which states that. If a coloum contains a leading entry then all entries below that leading entry are zero.

According to definition matrix B's coloumn contains leading entries as 1 and below that all entries are zero. So matrix B is in echelon form.

PART "C"

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$ is in reduced row echelon form:—

SOLUTION:-

As $C = \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 5/5 & 0/5 & 0/5 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $R1/5$

$C = \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

Yes, Matrix C is in reduced row echelon form Because Reduced row echelon form states that In reduced row echelon form the leading co-efficient must be 1. In each row is to the right of the leading co-efficient in the row above it. According to matrix C satifies the definition properties. So it is in reduced rows echelon form.

PART 'D'
$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$ in reduced Row echelon form:-

SOLUTION:—

$$\text{As } D = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

No, matrix D is not in Reduced row echelon form, because if a matrix is in reduced row echelon form then its rows (non-zero) contains. Its first entries as a number 1. Which is known as Leading 1 i-e the first non-zero entry is 1 Also if there are any rows contains only zero So it is not reduced row echelon form.

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Question No: 3

Part A:- The row echelon form is used to solve the system of linear equations. What is the difference b/w row echelon and reduced row echelon form. What is the practical use of reduced row echelon form give one example.

Ans:-

The echelon form of a matrix is not unique which means there are infinite answers possible when you perform Row reduction. Reduced Row echelon form is at the other end of the spectrum it is unique which means row reduction on a matrix will produce the same answer no matter how to perform the same row operations:-

WHAT IS ROW ECHELON FORM:-

A matrix is in Row echelon form if it meets the following requirments.

- The first non-zero number from the left (the "Leading coefficient") is always to the Right of the first non-zero number in the row above.

- Rows consisting of all zeros are at the bottom of the matrix.

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$$\begin{bmatrix} 1 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 2 & a_4 & a_5 \\ 0 & 0 & 0 & 1 & a_6 \end{bmatrix}$$

Row echelon form "a" can represent any number. Technically the leading co-efficient can be any number. However the majority Linear Algebra text books do state that the leading co-efficient must be number 1. To add to the confusion, Some definitions of row echelon state that there must be zeros both above and below the leading coefficient. It's therefore best to following the definition given in the text book you are following.

If the leading coefficient in each row is only non-zero number in that coloumn the matrix is said to be in reduced row echelon form

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

Row echelon forms are commonly encountered in Linear Algebra when you'll sometimes be asked to convert a matrix into this form the row echelon form can help us to see what a matrix represents and is also an important step to solving system linear equations.

What is REDUCED ROW ECHELON FORM:-

Reduced row echelon form is a type of matrix used to solve systems of linear equations. Reduced Row echelon form has four requirments

The first non-zero number in the first row (the leading entry) is the number 1

The second row also starts with the number 1 which is further to the right than the leading entry in the first row. For every subsequent row, the number 1 must be further to the right.

The leading entry in each row must be only non-zero number in its coloumn.

·Any non-zero rows are replaced at the bottom of the matrix

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$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

A 3x5 matrix in reduced Row Echelon Form

Q No:- 3 Part B

Find an echelon form for the below matrix using row operation. Where ID2 is 2nd digit of your ID is 12345 ID2 = 2, ID3 = 3 ID_First_last is the first and las digit of your ID:-

$$\begin{bmatrix} 1 & ID_2 & 8 \\ 2 & 8 & -1 \\ -ID_3 & 0 & 0 \\ 1 & -4 & ID\_first\_last \end{bmatrix}$$

My ID 5534

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 2 & 8 & -1 & 0 \\ -3 & 0 & 0 & 0 \\ 1 & -4 & 54 & 0 \end{array}\right] \times(-2)$$

$R_2 - 2 \times R_1 \rightarrow R_2$

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$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 0 & -2 & -17 & 0 \\ -3 & 0 & 0 & 0 \\ 1 & -4 & 54 & 0 \end{array}\right] \times(3) \sim R_3-(-3)\times R_1 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 0 & -2 & -17 & 0 \\ 0 & 15 & 24 & 0 \\ 1 & -4 & 54 & 0 \end{array}\right] x(-1) \sim R_4 - 1 \times R_1 \to R_4$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 0 & -2 & -17 & 0 \\ 0 & 15 & 24 & 0 \\ 0 & -9 & 46 & 0 \end{array}\right] \times\left[\frac{15}{2}\right] \sim R_3 - \left[\frac{-15}{2}\right]\times R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 0 & \boxed{-2} & -17 & 0 \\ 0 & 0 & \frac{-207}{2} & 0 \\ 0 & -9 & 46 & 0 \end{array}\right] \times\left(\frac{-9}{2}\right) \quad R_4-\left(\frac{9}{2}\right)\times R_2 \to R_4$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 0 & -2 & -17 & 0 \\ 0 & 0 & \boxed{-207/2} & 0 \\ 0 & 0 & \frac{245}{2} & 0 \end{array}\right] \times\left(\frac{245}{207}\right) \sim R_4-\left(\frac{-245}{207}\right)\times R_3 \to R_4$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 8 & 0 \\ 0 & -2 & -17 & 0 \\ 0 & 0 & \frac{-207}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + 5\times x_2 + 8\times x_3 = 0 \\ -2\times x_2 - 17\times x_3 = 0 \\ -\frac{207}{2}\times x_3 = 0 \end{cases} \quad (1)$$

Find the variable x_3 from eq 3 of the system (1)

$$-\frac{207}{2}\times x_3 = 0$$

$$\boxed{x_3 = 0}$$

Find the variable x_2 from eq 2 of the system (1).

$$-2\times x_2 = 17\times x_3 = 17\times 0 = 0$$

$$\boxed{x_2 = 0}$$

Find the variable x_1 from equation 1 of the system (1).

$$x_1 = -5\times x_2 - 8\times x_3 = -5\times 0 - 8\times 0 = 0$$

$$\boxed{x_1 = 0}$$

Answer

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

General Solution:—

$$X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$