# Course Details

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| Course Tittle | E.m.F |
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| Modul | 4th |

## Student Details

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Q No 1

Part 1(a)

Transform the vector $B = Yi(x+z)j$

Ans:-

$B = Yi(x+z)j$

Point $(-2, 6, 3)$

Then

$B = Yi(xj + zj)$

$B = yxij + yzij$

$\rho = \sqrt{x^2 + y^2}$

$= \sqrt{(-2)^2 + (6)^2} \Rightarrow \sqrt{4+36}$

$\Rightarrow \sqrt{40}$

$\boxed{\rho = 6.32}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \tan^{-1}\left(-\frac{6}{2}\right)$

$\boxed{\phi = -71.575}$

$\boxed{Z = 3}$

So

$B = \boxed{6.32, -71.56, 3}$

Part b

Point (3, 4, 5) from cartesion to spherical coordinates

Solution: P(3, 4, 5)

for spherical coordinate system we have to find

$r, \theta, \phi$

As we know

$r = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{3^2+4^2+5^2} \Rightarrow \sqrt{9+16+25}$

$r = \sqrt{50} \Rightarrow \boxed{7.07}$

As $\theta = \tan^{-1}(y/x)$

$= \tan^{-1}(4/3)$

$\theta = \tan^{-1}(1.33)$

$\theta = 53.1$

As $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{3^2+4^2}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right) \Rightarrow \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{5}{5}\right)$

$\phi = \tan^{-1} 1$

P.T.O

$$\phi = 45^\circ$$

$$r = 7.07 \quad , \quad \theta = 53.1 \quad , \quad \phi = 45^\circ$$

Part c

Find the spherical coordinate of
A (2, 3, -1)

Solution: $r, \theta, \phi$

As

$r = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{2^2+3^2+(-1)^2}$

$r = \sqrt{4+9+1} \Rightarrow \sqrt{14}$

$r = \boxed{3.74}$

As $\theta = \tan^{-1}(y/x)$

$\Rightarrow \tan^{-1}(3/2)$

$\theta \Rightarrow \tan^{-1}(1.5)$

$\theta = \boxed{56.3^\circ}$

As $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{+z}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{2^2+3^2}}{-1}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{4+9}}{-1}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{13}}{-1}\right)$

$\phi = \tan^{-1}(-3.60)$

$\phi = \boxed{74.4}$

$r = 3.74, \quad \theta = 56.3^\circ, \quad \phi = 74.4$

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Part (d)

Find the Cartesian coordinate of $B(4, 25, 120)$

Solution :-

B point is actully given is sperical

$(r, \theta, \phi)$

We have find $(x, y, z)$

$x = r \sin\theta \cos\phi$

$= 4 \sin 25 \cos 120$

$= 4(0.42)(-0.5)$

$\boxed{x = -0.84}$

As

$y = r \sin\theta \sin\phi$

$= 4 \sin 25 \sin 120$

$= 4(0.42)(0.86)$

$\boxed{y = 1.45}$

As

$z = r \cos\theta$

$= 4 \cos 25$

$= 4(0.906)$

$\boxed{z = 3.62}$

$(x, y, z,) =$ $x = -0.84$
$y = 1.45$
$z = 3.62$

Part e

Find the force b/w two charge when ----- charge are 2nc and -1nc in $\mu N$.

Given Data

$q_1 = 2nc$ , $q_2 = -1nc$

$d = 4cm$

Required

$F = ?$

Solution:

$$F = K \frac{q_1 q_2}{r^2} \rightarrow (1)$$

As

$$K = \frac{1}{4\pi \varepsilon_0}$$

Put value in Equation -> (1)

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$F = \frac{2\times10^{-9} \times -1\times10^{-9}}{4(3.14) \times 8.85\times10^{-12} \times (4\times10^{-2})^2}$$

$$F = -1.124\times10^{-5}$$

$$F = \boxed{-11.24\,\mu N}$$

Q No 1



Part f

Find the electric field intensity of two charge ---- -- by a distance 1m in air.

Given

$q = -2c$

$q = -1c$

$d = 1m$

Required

$E = ?$

Solution

$\therefore K = 9 \times 10^9$

$$E = \frac{Kq_1}{d^2}$$

$$E_1 = \frac{9 \times 10^9 \times (-2)}{(+1)^2}$$

$$E_1 = \frac{9 \times 10^9 \times (-2)}{1} \Rightarrow \boxed{-18 \times 10^9 \text{ V/m}}$$

Now

$$E_2 = \frac{Kq_2}{d^2}$$

$$E_2 = \frac{9 \times 10^9 \times (-1)}{(1)^2}$$

$$E_2 = \frac{9 \times 10^9 \times (-1)}{1} = \boxed{-9 \times 10^9 \text{ V/m}}$$

$$(E_1, E_2) = -18 \times 10^9 \text{ V/m}, \quad -9 \times 10^{-9} \frac{V}{m}$$

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Part 8

Determine the charge that produce ----- ------ -- a distance 30cm in vacuum (in $10^{-8}$c)

Given Data

$$E = 40 \text{ V/cm}$$

$$d = 30 \text{ cm}$$

Required

$$Q = ?$$

Solution:

$$E = \frac{KQ}{d^2}$$

$$Ed^2 = KQ$$

Divide both side by K

$$\frac{Ed^2}{K} = \frac{\cancel{K}Q}{\cancel{K}}$$

$$\frac{Ed^2}{K} = Q \quad \text{----} \quad (E)$$

Now putting the value in Equation

$$Q = \frac{Ed^2}{K}$$

$$Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$Q = \frac{40 \times 900}{9 \times 10^9}$$

$$Q = 4 \times 10^{-6} \text{ C}$$

OR

$$Q = 4 \mu C$$

Part h

A Charge of $2\times10^{-7}$ is acted upon by a force ---- ----- $4.5\times10^{-7}$ both the charge are in vacuum.

Given

$$q_1 = 2\times10^{-7}c$$
$$q_2 = 4.5\times10^{-7}c$$
$$F = 0.1N$$
$$k = 9\times10^{9}$$

Required

$$d = ?$$

Sol

Using Formula

$$F = K\frac{q_1 q_2}{d^2}$$

$$d^2 = K\frac{q_1 q_2}{F} \quad --- \text{(E)}$$

Putting value in Equation

$$d^2 = \frac{9\times10^{9}(2\times10^{-7})(4.5\times10^{-7})}{0.1}$$

$$d^2 = 8.1\times10^{-3} \Rightarrow 0.0081$$

$$d^2 = 0.0081$$

Taking ~~cuod~~ Square Root on both side

$$\sqrt{d^2} = \sqrt{0.0081}$$

$$d = 0.09m$$

$$d = 9\times10^{-2}\,m$$ Ans

Part (a)

Find the angle B/w the vectors shown in figure.

Given that

$$\bar{A} = \sqrt{3}\,i_x + i_y$$

$$\bar{B} = 2i_x$$

$$\bar{A}\cdot\bar{B} = 2\sqrt{3}$$

Required

$$\theta = ?$$

Since we know that

$$A\cdot B = |A||B|\cos\theta_{AB}$$

$$|A| = \sqrt{(\sqrt{3})^2 + (1)^2} \Rightarrow \cancel{\sqrt{9}}$$

$$|A| = \sqrt{3+1} \Rightarrow \sqrt{4} \Rightarrow 2$$

$$|B| = \sqrt{(2)^2} = \sqrt{4} \Rightarrow 2$$

$$A\cdot B = |A||B|\cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{A\cdot B}{|A||B|}$$

$$\theta_{AB} = \cos^{-1}\frac{A\cdot B}{|A||B|}$$

$$= \cos^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{2}\times 2}\right) \Rightarrow \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta_{AB} = 30^\circ$$

Part (b)

Find the gradient of each - - - - -
- - - are constant.

(i) $f = ax^2 + by^3z$

Solution:

As we know that

$$\nabla f = \text{grad} f$$

$$\nabla f = \left(\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k\right)(ax^2 + by^3z)$$

$$\nabla f = \frac{\partial}{\partial x}(ax^2+by^3z)i + \frac{\partial}{\partial y}(ax^2+by^3z)j + \frac{\partial}{\partial z}(ax^2+by^3z)k$$

$$\nabla f = \frac{\partial}{\partial x} ax^2 i + \frac{\partial}{\partial y} by^3z j + \frac{\partial}{\partial z} by^3z k$$

$$\nabla f = 2axi + 3by^2z j + by^3 k$$

Hence grad f is $2axi + 3by^2zj + by^3k$

where a & b are constant.

(ii) $f = ar^2 \sin\phi + brz\cos 2\phi$

Sol:- $\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$

So

$$\nabla f = \frac{\partial}{\partial r}(ar^2\sin\phi + brz\cos 2\phi)\hat{r} +$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}(ar^2\sin\phi + brz\cos 2\phi)\hat{\theta} +$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(ar^2\sin\phi + brz\cos 2\phi)\hat{\phi}$$

Take partial derivative

$$\Rightarrow \nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r}(0) +$$

$$\frac{1}{r\sin\theta}(a\cancel{r}^2\cos\phi - 2brz\sin 2\phi)\phi$$

$$\Rightarrow \nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} +$$

$$\frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}$$

Now Gradient for cylindrical

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\Rightarrow \nabla f = \frac{\partial}{\partial \rho}(ar^2\sin\phi + brz\cos 2\phi)\hat{\rho} +$$

$$\frac{1}{\rho}\frac{\partial}{\partial \phi}(ar^2\sin\phi + brz\cos 2\phi)\hat{\phi} +$$

$$\frac{\partial}{\partial z}(ar^2\sin\phi + brz\cos 2\phi)\hat{z}$$

Take partial derivative :
Then the first term become zero

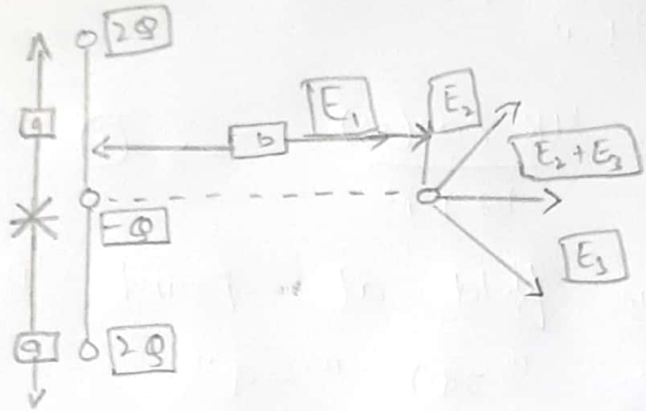
$$\nabla f = \frac{1}{\rho}\left(ar^2\cos\phi - 2brz\sin\phi\right)\hat{\phi} + \left(br\cos 2\phi\right)\hat{z}$$

So

$$\nabla f = \frac{1}{\rho}\left(ar^2\cos\phi - 2brz\sin\phi\right)\hat{\phi} + \left(br\cos 2\phi\right)\hat{z}$$

Q NO (3)

Three Pointer charge are placed on the y-axis . - - - - - - - at point P on the x-axis.




Sol :-

The distance b/w charge 2q & Poin 'p'

$$r^2 = b^2 + a^2$$

So

$$r = \sqrt{b^2+a^2}$$

Assume that charge 2q make angle ($\alpha$) & ($-\alpha$) with x-axis

Magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{Kq}{r^2}$

$$= \frac{K(2q)}{r^2}$$

$$= \frac{K(2q)}{b^2+a^2}$$

So Resultant of $\vec{E_1}$ & $\vec{E_2}$ is

$$\vec{E}_{1+2} = \vec{E_1} + \vec{E_2} = \vec{E}_{1x} + \vec{E}_{2x}$$

(Y - component will be cancel)

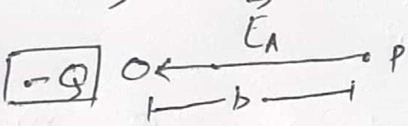
$$= \frac{K(2Q)}{b^2+a^2}\left(\cos(\alpha) + \cos(-\alpha)\right)$$

$$= \frac{K(2Q)}{b^2+a^2}\left(2\cos(\alpha)\right) \quad \because \cos(\alpha) = \cos(-\alpha)$$

$$\vec{E}_{1+2} = \frac{4K\cos(\alpha)}{b^2+a^2} \rightarrow (i)$$

-> Electric field at point 'p' due to charge ~~(-Q)~~ "-Q"

-> As charge is -ive Electric field at point will to directed toward charge "-Q".




$$\vec{E}_A = \frac{-K(Q)}{b^2}$$

Net electric field at point 'p' will be

$$\vec{E}_{net} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$$

$$= \frac{-K(Q)}{b^2} + \frac{4KQ\cos\alpha}{b^2+a^2}$$

$$= \frac{-KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $K = 9\times10^9\ Nm^2/C^2$

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$$\vec{E}_{Net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

Now

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

So

$$\vec{E}_{net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right] - (a^2+b^2)\right]$$