Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Mid – Term Examinationspring2020
Date:

Course Code: MTH 102		**Course Title:**	Calculus and analytic geometry
Prerequisite:		**Instructor:**	HIMAYATULLAH
Module: 3	**Program:** BEE	**Total Marks:** 30	**Time Allowed:** 90 min

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	. **Identify** $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y=\left(x+\frac{1}{x}\right)(x-\frac{1}{x}+1)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have nay horizontal tangent if so where ?	Marks 10 CLO1 C1

Q 1 (a) :- Identify $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

Sol :-

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$\lim_{h \to 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h[\sqrt{2+h} + \sqrt{2}]}$$

$$\lim_{h \to 0} \frac{\cancel{2} + \cancel{h} - \cancel{2}}{\cancel{h}[\sqrt{2+h} + \sqrt{2}]}$$

$$\lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

$$\frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$ Ans

Q 1 :- (b)

Sol :- $y = \left(x + \frac{1}{x}\right) \cdot \left(x - \frac{1}{x} + 1\right)$

Applying $\frac{d}{dx}$ on b.s

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)\right]$$

Product rule

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)\frac{d}{dx}\left(x-\frac{1}{x}+1\right) + \left(x-\frac{1}{x}+1\right)\frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$= \left(x+\frac{1}{x}\right)\left(\frac{d}{dx}x - \frac{d}{x}\frac{1}{x} + \frac{d}{dx}1\right) + \left(x-\frac{1}{x}+1\right)$$

$$\left(\frac{d}{dx}x + \frac{d}{dx}\frac{1}{x}\right)$$

(3)

$$y' = \left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right)+\left(x-\frac{1}{x}+1\right)\left(1-\frac{1}{x^2}\right)$$

$$y' = \left[x+\cancel{\frac{x}{x^2}}+\frac{1}{x}+\frac{1}{x^3}\right]+\left[x-\cancel{\frac{x}{x^2}}-\frac{1}{x}+\frac{1}{x^3}+1-\frac{1}{x^2}\right]$$

$$y' = \left[x+\frac{1}{x}+\frac{1}{x}+\frac{1}{x^3}\right]+\left[x-\frac{1}{x}-\frac{1}{x}+\frac{1}{x^3}-\frac{1}{x^2}+1\right]$$

$$y' = \left[x+\frac{2}{x}+\frac{1}{x^3}\right]+\left[x-\frac{2}{x}-\frac{1}{x^2}+\frac{1}{x^3}+1\right]$$

$$y' = x+\frac{2}{x}+\frac{1}{x^3}+x-\frac{2}{x}-\frac{1}{x^2}+\frac{1}{x^3}+1$$

$$y' = x+x-\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^3}+1$$

$$y' = 2x-\frac{1}{x^2}+\frac{2}{x^3}+1$$

Q2:- (a)

Sol:-

(i) Since $s = 160t - 16t^2$ $\boxed{16 \times 2t}$

$$v = \frac{ds}{dt} = 160 - 32t$$

At max hight $v = 0$

$$160 - 32t = 0 \Rightarrow t = \frac{160}{32}$$

$$t = 5 \text{ sec}$$

$$S_{max} = S_{(5)} = 160 \times 5 - 16(5)^2$$

$$= 400ft$$

Velocity and speed when it is 256ft high:

256ft into meters:

$$1ft = 0.3\,m$$

$$256 = 0.3 \times 256\,m$$

$$256 = 76.8\,m$$

$$S = 160t - 16t^2$$

$$76.8 = 160t - 16t^2$$

$$16t^2 - 160 + 76.8 = 0$$

$$t_1 = 9.49s$$

$$t_2 = 0.5s$$

Calculate velocity at $t_2 = 0.5s$

$$\frac{ds}{dt} = v = \frac{d(160t - 16t^2)}{dt}$$

$$v = 160 - 32t$$

For that velocity put $t = t_2 = 0.5s$

$$v = 160 - 32(0.5)$$

$$v = 144 \, m/s.$$

Since

$$v = 160 - 32t$$

$$a = \frac{dv}{dt}$$

$$= 0 - 32$$

$$= -32 \, m/sec$$

Q3 :- (a)

Sol :-

As

$$y = x^4 - 2x^2 + 2$$

Let

$$y = f(x)$$

So

$$f(x) = x^4 - 2x^2 + 2$$

Find deravaties

$$\frac{d}{dx} f(x) = \frac{d}{dx} (x^4 - 2x^2 + 2)$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} x^4 - \frac{d}{dx} 2x^2 + \frac{d}{dx} 2$$

$$f'(x) = 4x^3 - 4x + 0$$

Now let

$$f'(x) = 0$$

So

$$4x^3 - 4x = 0$$

$$4x (x^2 - 1) = 0$$

So

$$4x = 0$$

b.s divided by 4

$$\frac{4x}{4} = \frac{0}{4}$$

$$\boxed{x = 0} \quad \text{---} \quad (1)$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$\boxed{x = -1} \quad \text{---} \quad (2)$$

$$\boxed{x = +1} \quad \text{---} \quad (3)$$

So

$$x = \{0, 1, -1\}$$

Now as we know that

$$f(x) = x^4 - 2x^2 + 2$$

put $x = 0$

$$f(0) = (0)^4 - 2(0)^2 + 2$$

$$f(0) = 0 - 0 + 2$$

$$f(0) = 2$$

Now put $x = 1$

So

$f(1) = x^4 - 2x^2 + 2$

$f(1) = (1)^4 - 2(1)^2 + 2$

$f(1) = 1 - \cancel{2} + \cancel{2}$

$\boxed{f(1) = 1}$

Now put $x = -1$

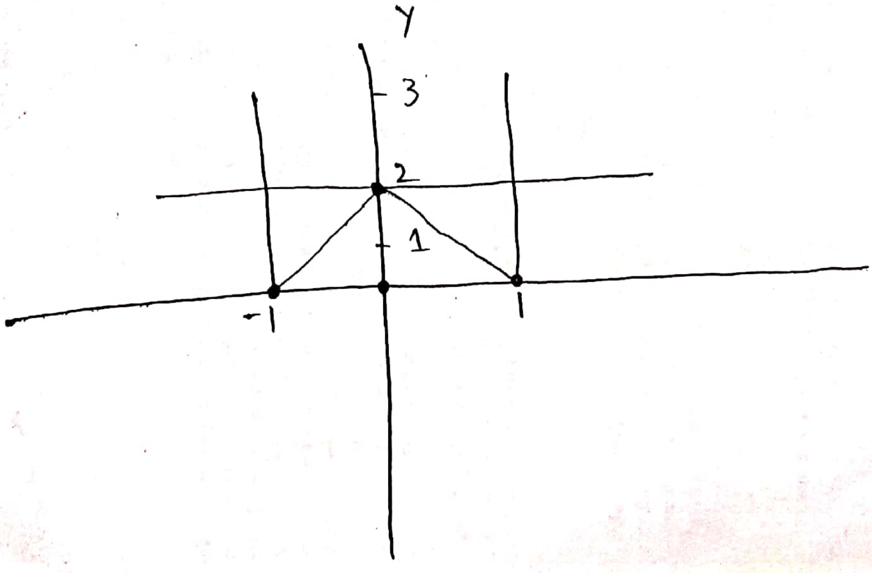
$f(-1) = x^4 - 2x^2 + 2$

$f(-1) = (-1)^4 - 2(-1)^2 + 2$

$f(-1) = 1 - 2(1) + 2$

$f(-1) = 1 - \cancel{2} + \cancel{2}$

$\boxed{f(-1) = 1}$



(9)

So the horizontal lies on

$f(x) = x^4 - 2x^2 + 2$ are

$(1, 1)$ $(-1, 2)$ $(0, 2)$ Ans.