

Date: _____

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Answer No 1:

$$\text{Solution: } v_1 = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$$

$$= 1D_1(6 \cdot 0 \cdot 3) + 1D_2(6 \cdot 0 \cdot 3) + 1D_3(6 \cdot 0 \cdot 3)$$

The desired candidates of R_3 is
(1.6, 6.6, 0.6)

Similarly

(1.0, 6.0, 0.0)

and (1.3, 6.3, 0.3).

$$\text{So } A = \begin{bmatrix} 1.6 & 1.0 & 1.3 \\ 6.6 & 6.0 & 6.3 \\ 0.6 & 0.0 & 0.3 \end{bmatrix}$$

So you can see each column of above R_3 vectors are linearly dependent.

2)

Date: _____

Answer No 2:-

In matrix form we can write as:

$$450x_1 + 250x_2 = 1000$$

$$400x_1 + 350x_2 = 500$$

$$\begin{bmatrix} 450 & 250 \\ 400 & 350 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

$$A \quad X \quad B$$

$$\Rightarrow X = A^{-1} \cdot B$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 450 & 250 \\ 400 & 350 \end{bmatrix}^{-1} \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 350 & -250 \\ -400 & 450 \end{bmatrix}, \quad |A| = \begin{vmatrix} 450 & 250 \\ 400 & 350 \end{vmatrix}$$

$$|A| = (450)(350) - (250)(400) = 157,500 - 100,000 \\ = 57,500$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{57500} \begin{bmatrix} 350 & -250 \\ -400 & 450 \end{bmatrix} \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{57500} \begin{bmatrix} 350000 - 125000 \\ -400000 + 225000 \end{bmatrix} = \begin{bmatrix} 225000 \\ -175000 \end{bmatrix}$$

$$x_1 = 225000, \quad x_2 = 175000$$

$$\text{Total cost} = x_1 + x_2 = 400000$$

3)

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Answer No 3:-

A vector space is a collection of objects called vectors, which may be added together and multiplied (scaled) by numbers scalars are often taken to be real numbers but there are also vector spaces with scalar multiplication by complex numbers rational numbers or generally any field.

A)

solution: $\text{tr} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix}$ for $k \in \mathbb{R}$

according to the def. of vector space if any vector space then it will become vector space so in this case $\begin{pmatrix} ka & b \\ kc & d \end{pmatrix}$ is not a vector space

ie $k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

B) Solution:-

Let $p(x) = a_1 x^3 + a_2 x^2 + x + c$

which is defined and correct according to the definition of vector space.

4)

Date: _____

Answer No 4'r

$$\text{let } M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2 \times 2}$$

(a) $|M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
 if we take inverse of M i.e. $M^{-1} = \frac{\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}{ad - bc}$

$$\text{i.e. } M \cdot M^{-1} = I$$

(b) All 2×2 identity matrices are:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

and $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(c) All 2×2 matrices for zero det. are:

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

(d)

$$\text{Det } A = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 0 & 6 \\ 3 & 1 & 0 \end{vmatrix} \quad \text{Eliminate } R_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 6 & 0 & 6 \\ 3 & 1 & 0 \end{vmatrix} = 1 \cdot 0 \cdot 0 + 1 \cdot 6 \cdot 3 + 1 \cdot 6 \cdot 1 - 1 \cdot 0 \cdot 3 - 1 \cdot 6 \cdot 0 - 1 \cdot 6 \cdot 1 =$$

$$0 + 18 + 6 - 0 - 0 - 6 = \boxed{18}$$