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Subject : Linear Algebra

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Date: 20-08-2020

Q1 (a) Given:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

Find:

Identify the (3,2) entry AB

Sol:

As we know that.

$row_3(A) \cdot col_2(B)$

$$= [0, 1, -2] \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} = -5$$

Q1(b) Given:

interpolate the point
(1,3) (2,4) (3,4)

Req:

Find the quadratic polynomial.

Sol:

As we know that.

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$
$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now

$(x_1, y_1) = (1,3)$ $(x_2, y_2) = (2,4)$

$(x_3, y_3) = (3,7)$ put in above.

$a_2 + a_1 + a_0 = 3$

$4a_2 + 2a_1 + a_0 = 4$

$9a_2 + 3a_1 + a_0 = 7$

$$A_b = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array}\right]$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array}\right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\overset{R}{\sim} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 - 3R_2$$

so $a_2 + a_1 + a_0 = 3$ —(1)

$-2a_1 - 3a_0 = -8$ —(2)

$a_0 = 4$ put in 2.

$-2a_1 - 12 = -8$ $a_1 = \frac{4}{2} = 2$.

put in 1

$a_2 - 2 + 4 = 6$

$a_2 = 1$

Q2(a) Given:

$A = 2 \quad B = -3$

Find: $|A^{-1} B^{T}|$

Sol:

As we know that.

Since $|A^{-1} B^{t}| = |A^{-1}| \, |B^{t}|$

$= \frac{1}{|A|} |B| \qquad |B^{t}| = |B|$

so $|A^{-1} B^{t}| = \frac{1}{|A|} |B|$

$= \frac{1}{2} \cdot 3 = 3/2$

Q2 (b) Given:

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

Find:

Solve the linear system of equation:

Sol.

As we know that.

$$Ab = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 3 & -1 & 1 & 3 \end{array}\right] R_2 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 3 & 1 & 1 & 3 \\ 0 & -3 & -1 & -4 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -2 & -5 & 6 \\ 0 & -3 & -1 & -4 \end{array}\right] R_2 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -2 & -5 & -6 \\ 0 & 0 & 13 & -26 \end{array}\right] 2R_3 - 3R_2$$

$13z = -26$

$z = \frac{-26}{13}$

$[z = -2]$

$-2y - 5z = -6$

put $z = -2$.

$-2y - 5(-2) = 6$

$-2y + 10 = 6$

$-2y = 6 - 10$

$-2y = -4$

$y = \frac{\cancel{+4}\,2}{\cancel{+2}\,1}$

$[y = 2]$

$x + y + 2z = -1$

$x + 2 + 2(-2) = -1$

$x + 2 - 4 = -1$

$x - 2 = -1$

$x = -1 + 2$ $[x = 1]$

Q3

Given:

$$A=\begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

Req:

$$A^{-1}$$

Sol:

As we know that

$$A=\begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

$$= 3\begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-4-6)+2(-15-2)+1(0-6)$$

$$|A| = -94$$

Now

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 0 & 3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 0 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A.$$

$$A^{-1} = \frac{1}{-94}\begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix} \underline{\text{Ans}}.$$