Q1 Find PQ where P is the point in three-dimensional space with co-ordinates (4, 1, 3) & and the point Q with co-ordinates (1, 2, 4). Find the distance between PQ in the ratio 1:3.

Sol:-

co-ordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3k$.

or

$$OQ = \vec{OQ} - \vec{OP}$$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \rightarrow 1$$

Now distance between P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \rightarrow (2)$$

Let M be the point which divided PQ in ratio 1:3, then by ratio theorem position vector of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}.$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}.$$

$$= \frac{13\hat{i} + 5\hat{j} + 13k}{4} \rightarrow (3).$$

Here eq(i), (ii) and (iii) are the required solution.

Q No 8//

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx.$$

Sol:-

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx.$$

$$
\begin{array}{r|l}
 & 2x-1 \\
\hline
2x^2+x & 4x^3+10x+4 \\
 & \underline{\pm 4x^3 \quad \pm 2x^2} \\
 & -2x^2+10x+4 \\
 & \underline{\mp 2x^2 \quad \mp x} \\
 & 11x+4.
\end{array}
$$

So

$$2x-1+\frac{11x+4}{2x^2+x}=\frac{4x^3+10x+4}{2x^2+x}.$$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1 + \int \frac{11x+4}{2x^2+x} \rightarrow (1).$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+2}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \rightarrow 2$$

Now Find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)}$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1) + Bx \rightarrow 3$$

Put $x=0$ in eq 3

$$11(0)+4 = A[2(0)+1] + B(0)$$

$$4 = A$$

OR

$$\boxed{A = 4}$$

Now put $x = -\frac{1}{2}$ in 3

$$11(-\tfrac{1}{2})+4 = B(-\tfrac{1}{2}).$$

$$-\frac{11}{2}+4 = -\frac{B}{2}$$

$$\frac{-11+8}{2} = \frac{-B}{2}$$

$$-3 = -B \Rightarrow \boxed{B=3}$$

put the values of A & B in A.

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integration on b.sides:

$$\int \frac{11x+4}{x(2x+1)}dx = \int \frac{4}{x}dx + \int \frac{3}{2x+1}dx.$$

$$= 4\int \frac{1}{x}dx + 3\int \frac{1}{2x+1}dx.$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

putting those values in (2)

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln(2x+1).$$

Now put all those values in eq (i)

$$\int \frac{4x^3+10x+4}{2x^2+x}dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Q3

(a) $\int_0^2 x^2 e^x dx$

sol:-

$\int_0^2 x^2 e^x dx.$

$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2\right) dx.$

$= x^2 e^x - 2\int x e^x dx.$

$= x^2 e^x - 2\left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x\right) dx\right]$

$= x^2 e^x - 2\left[x e^x - \int e^x dx\right].$

$= x^2 e^x - 2x e^x + 2e^x.$

Now put limits

$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$= (2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)).$

$= (4e^2 - 4e^2 - 2e^2 - 2)$

$= (2e^2 - 2).$

(b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx.$

First find integration

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \to (1).$

Let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$2dy = \frac{1}{\sqrt{x}}\, dx \to$ put in (1)

③

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put

$$y = \sqrt{x}$$

$$= -2\cos\sqrt{x}$$

Put limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos\ )$$

$$= -2\cos\sqrt{2} + 2\cos(1) \text{ Ans}$$

OR

$$= \cancel{2\cos\sqrt{2} - 2}$$

$$= 2\cos(1) - 2\cos\sqrt{2}$$

①

Q4 The laplace equation in 3.d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

so

$$u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$u(x, y, z) = (x^2+y^2+z)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -[x(-\tfrac{3}{2})(x^2+y^2+z^2)^{-5/3}(2x) + (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \to (1)$$

NOW

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -[y(-\tfrac{3}{2})(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \to (2)$$

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$\to 3$

Put Eq (1) (2) & (3) in (A)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2) + 3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}-\cancel{x^2}-y^2-z^2+3y^2-x^2-y^2-z^2+3z^2-x^2-y^2-z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0.$$

so the given $u(x,y,z)$ is sol of laplace equation.