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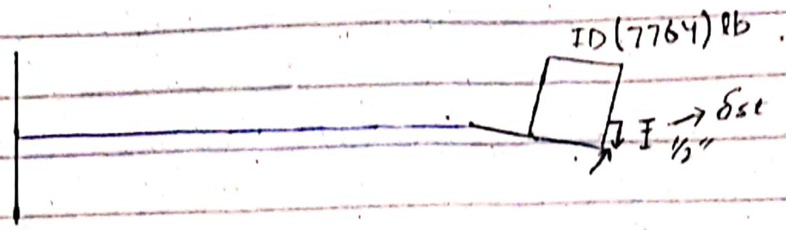
Sec	A
ID	7764
Subject	Earthquake
Submitted to	Engr. Yaseen Mehmood
Date	29/06/2020

Q No. (01)



Solution:-

The general EOM for SDOF system is

$Ku + C\dot{u} + m\ddot{u} = P(t)$

In our case system is undamped (c=0) undergoing free vibration (P(t)=0)

Hence general EOM becomes $Ku + m\ddot{u} = 0$ (1)

$K = 3EI/L^3$

$$K = \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$K = 7.55\ k/in$

In order to eliminate the chance of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec.

$K = 7.55\ k/in = 90625\ lb/ft$

$$m = \frac{7764\ lb}{32.2\ ft/sec^2}$$

$$m = 241.11\ slug$$

$$w_n = \sqrt{k/m}$$

$$w_n = \sqrt{\frac{90625}{241.11}}$$

$$w_n = 19.38\ rad/sec$$

$$T_n = 2\pi/w_n = 2\pi/19.38$$

$$T_n = 0.324\ sec.$$

Substituting the corresponding values in eq-(1)

$$90625\ u + 241.11\ \ddot{u} = 0$$

where 'k' is in lb/ft and 'm' is in lb sec/ft².

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0) \cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$$

$u(0) = 1/2'' = 1/24$ ft an $\dot{u}(0) = 0$

$u(t) = \left((1/24) \times \cos(19.38t) + 0 \right)$

$u(t) = (1/24) \times \cos(19.38t)$

Equivalent static force at any time "t" is

$f_s(t) = k \cdot u(t)$

$$f_s(t) = \frac{90625 \times \cos(19.38 ft)}{24}$$

$f_s(t) = 3776.04 \times \cos(19.38 ft)$

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Amplitude of dynamic displacement u_0 for undamped free vibration is.

$$u_0 = \sqrt{\left((u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right)}$$

$$u_0 = \sqrt{\left(\tfrac{1}{24}\right)^2 + 0}$$

$$u_0 = \tfrac{1}{24}\ ft$$

Aplitude of equivalent static force

$$ku_0 = 90625 \times \tfrac{1}{24}$$

$$ku_0 = 3776.04\ lb$$

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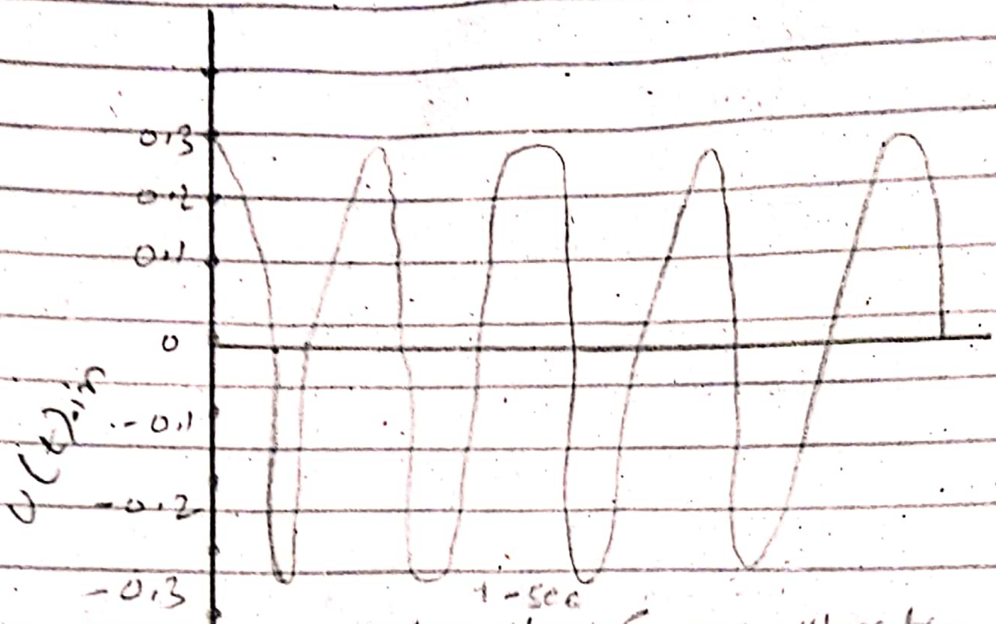
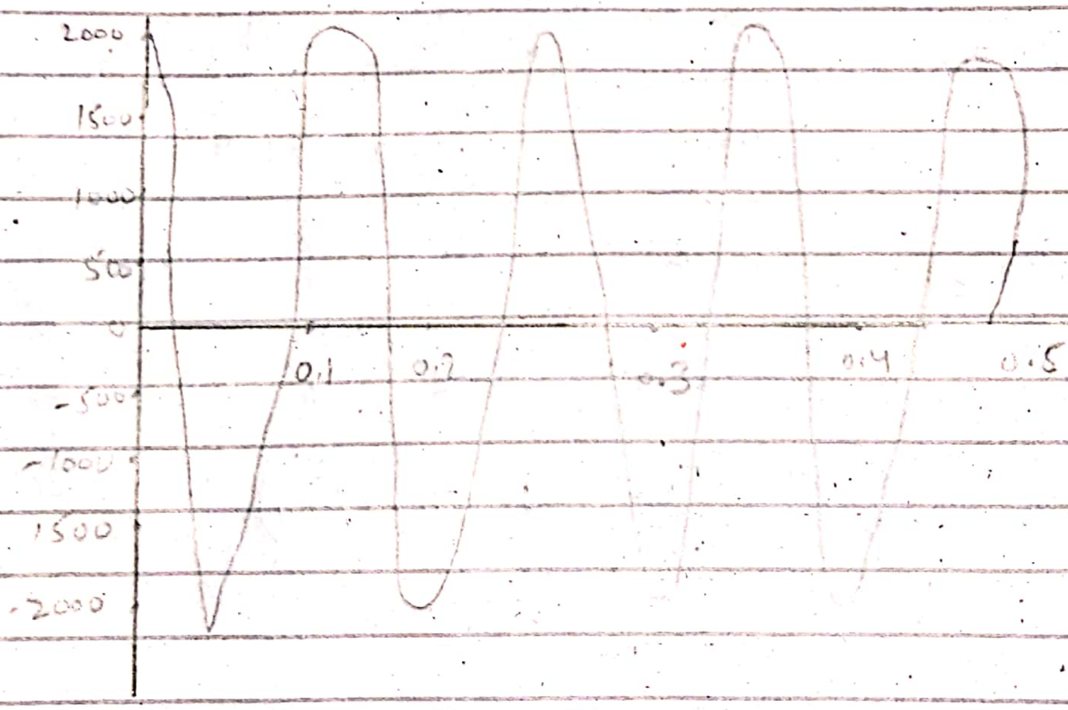


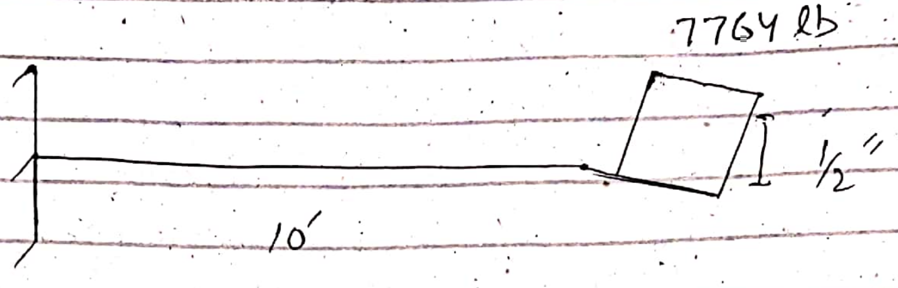
Undamped force vibration



Undamped Free vibration

Q No. (2)

Solution.



* EOM for damped free vibration is;

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{--- (i)}$$

* It is known from (ques 1)

$k = 90625$ lb/ft and $m = 241.11$ slug

$\omega_n = 19.38$

→ $C = \zeta \times 2m\omega_n = 2 \times 241.11 \times 19.38 \times \zeta$

($\zeta = 0.03 - 0.05$ with considerable cracking the damping ratio)

⇒ $C = 2 \times 241.11 \times 19.38 \times 0.05$

$C = 467.27$ sec/ft

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* By substituting values of K, C and m in eq (1) we get

$$90625u + 467.27\dot{u} + 241.11\ddot{u} = 0$$

* Solution to the E.O.M for damped free vibration is;

$$u(t) = e^{-\xi w_n t}\left(u(0)\cos(w_D t) + \frac{1}{w_D}\left(\dot{u}(0) + u(0)\,\xi w_n\right).\sin(w_D t)\right)$$

$w_D = 19.38$ rad/sec

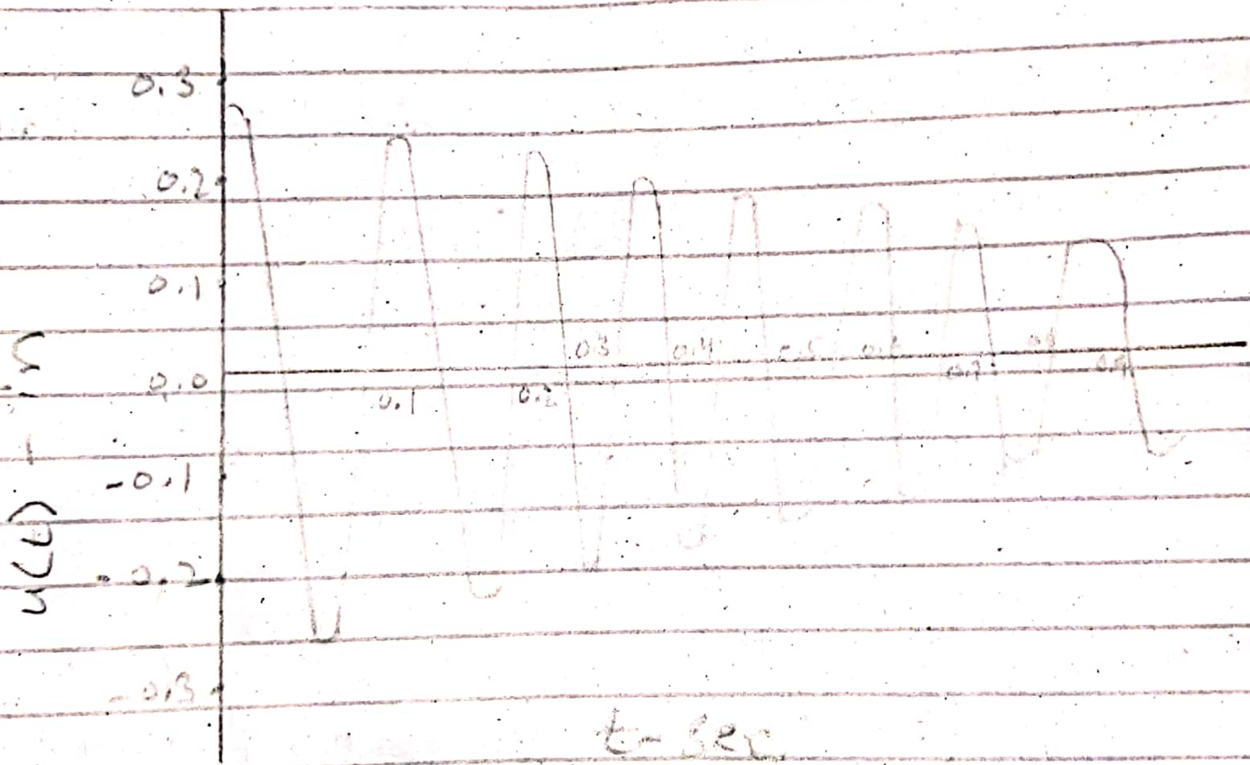
$$u(t) = e^{-0.05\times 19.38t}\left(\frac{1}{24}\times\cos(19.38t) + \frac{1}{19.38}\times\left(0 + \frac{1}{24}\times 0.05\times 19.38\times\sin(19.38t)\right)\right)$$

$$u(t) = e^{-0.969t}\left(0.041\times\cos(19.38t) + 0.051\times 0.041\times\sin(19.38t)\right)$$

$$u(t) = e^{-0.969t}\left(0.041\times\cos(19.38t) + 0.002\times\sin(19.38t)\right)$$

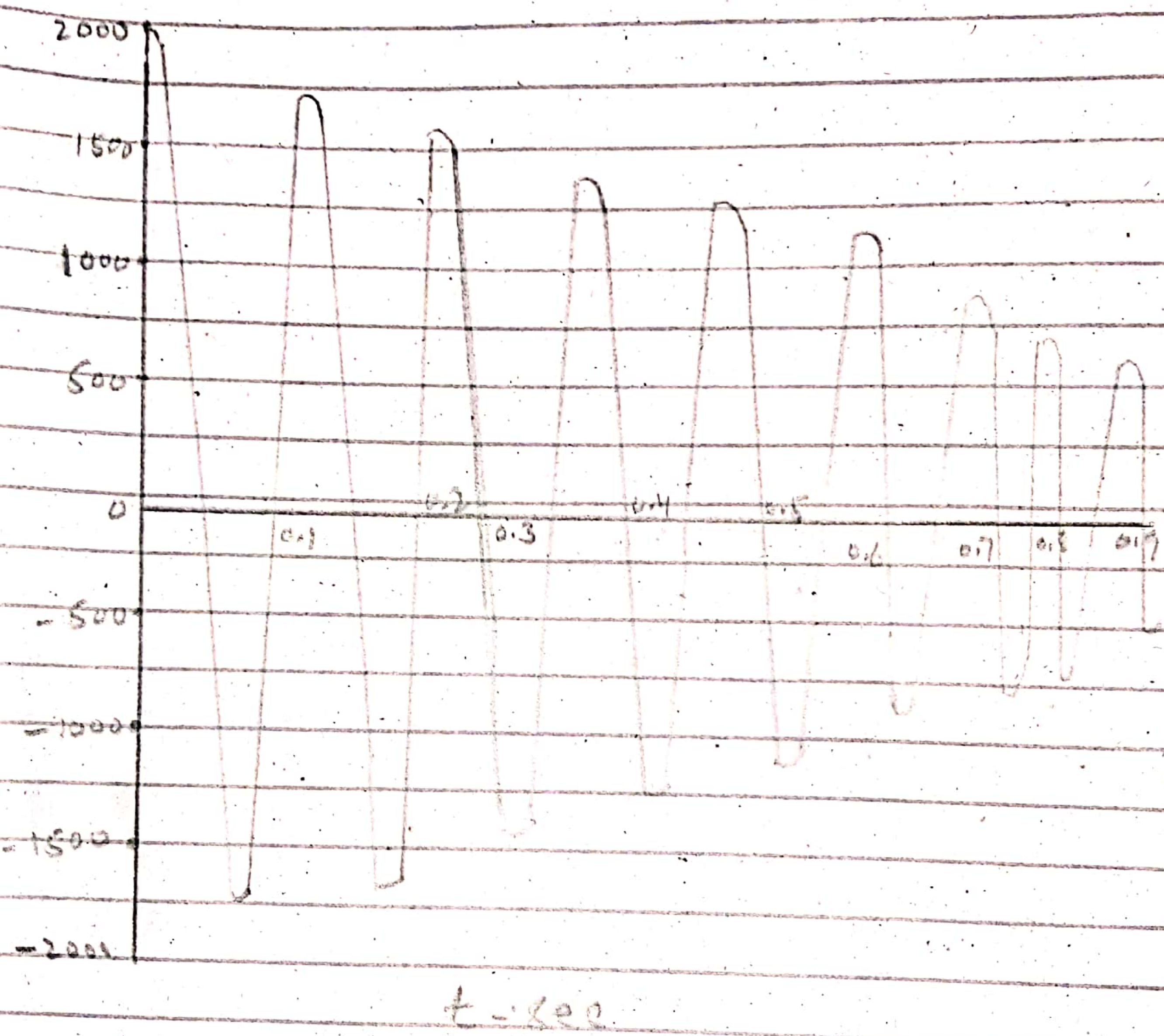
$$f_s(t) = K.u(t) = 90625\times u(t).$$

$$f_s(t) = e^{-0.969t}\left(3779.1\cos(19.38t) + 181.25 \times \sin(19.38t)\right)$$



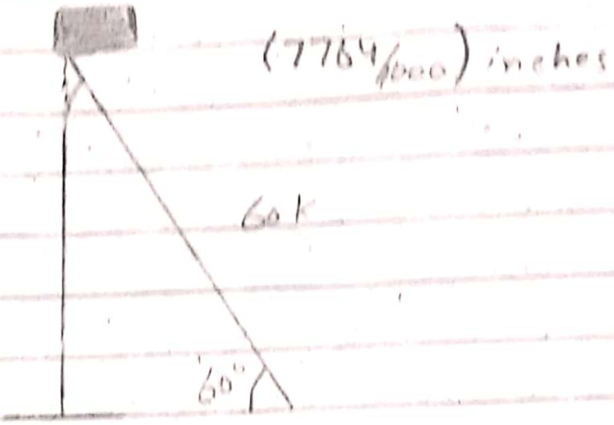
variation of displacement with time.

Damped Free vibration



variation of equvalent static force with time.

Q No. (03)



Solution:

$u_1 = 7764/1000 = 7.764 = 7.77$

After $j = 7$, $u_{j+1} = u_6 = 2.286\,cm$
$= 0.9''$

a): ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$7 = \frac{1}{2\pi\zeta} \ln(7.77/0.9)$$

$\zeta = 0.049 = 4.9\%$

b) $T_n = ?$

7 cycles of vibrations are completed in 3.57 sec.

Time required to complete one cycle = 3.57/7 = T_D

$T_D = 0.51 sec$

Now:

$\omega_D = \omega_n \sqrt{(1-\xi^2)}$

$2\pi/\omega_D = 2\pi / (\omega_n\sqrt{1-\xi^2})$

$\Rightarrow T_D = \dfrac{T_n}{\sqrt{1-\xi^2}}$

$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$

$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.049)^2}$

$\Rightarrow T_n = 0.5094 = 0.51 sec$

c) $K = ?$

$K = \dfrac{60 \times \cos 60^\circ}{7.77} = 3.8 \; k/in$

$K = 46332 \; lb/ft$

d). weight of the tank, $w = ?$

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{K}{(w/g)}} = \sqrt{\frac{K \times g}{w}}$$

$\Rightarrow \omega_n^2 = K \times g / w$

$\Rightarrow w = K \times g / \omega_n^2$

Also

$\omega_n = 2\pi / T_n$

$$w = \frac{46800 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$w = 9928.5$ lb

$w = 9.93$ k

e) $c = ?$

It is known that $\xi = \frac{c}{2m\omega_n}$

$\Rightarrow c = \xi \times 2m\omega_n = \xi \times 2m \times (2\pi / T_n)$

$$c = 0.049 \times 2 \times 2 \times (\pi/0.51)\left(\frac{9928.5}{32.2}\right)$$

$\Rightarrow c = 372.27$ lb.sec/ft

f) No. of cycles to reduce displacement amplitude from 7.77 in to 0.5"; $j = ?$

$$j = \frac{1}{2\pi\delta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow j = \frac{1}{2\times\pi\times0.049} \ln\left(\frac{7.77}{0.5}\right)$$

$$\Rightarrow j = 8.85 \text{ or } 9 \text{ cycles.}$$