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DIFFERENTIAL EQUATION

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Question # 1

(a)- $y' = (x+2)\,y^2$

Solution:

$$y' = (x+2)\,y^2$$

$$dy/dx = (x+2)\,y^2$$

$$\int 1/y^2\,dy = \int (x+2)\,dx$$

$$\int y^{-2}\,dy = \int (x+2)\,dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + c$$

$$y^{-1}/{-1} = x^2/2 + 2x + c$$

Multiplying both sides by -1.

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + c\right)$$

$$y = -\left(\frac{1}{x^2/2 + 2c + c}\right)$$

Answer.

Question # 1

(b) - $y' = (y + 9x)^2 \longrightarrow (i)$

Solution:

Let $y + 9x = u$

$$dy/dx + 9 = \frac{du}{dx}$$

$$dy/dx = \frac{du}{dx} - 9$$

So eq (i) become.

$$du/dx - 9 = u^2$$

$$du/dx = u^2 + 9$$

$$\int \frac{1}{u^2+9}\, du = \int dx$$

$$\int \frac{1}{(3)^2 + (u)^2}\, du = \int dx$$

$$1/3 \tan^{-1}\left(\frac{u}{3}\right) = x + C_1$$

Answer.

Question # 2.

$$x^3 dx + y^3 dy = 0$$

Solution:

Let, $x^3 = M$ & $y^3 = N$

So,

$$\frac{\partial M}{\partial y} = 0 \quad , \quad \frac{\partial N}{\partial x} = 0$$

Now equation will become,

$$u = \int M dx + k(y)$$

$$u = \int x^3 dx + ky \longrightarrow (i)$$

Now

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{dy} ky$$

But,

$$\frac{\partial u}{\partial y} = N$$ So,

$$N = d/dy \; ky$$

$$\int d \, k(y) = \int y^3 dy$$

$$k(y) = y^4/4 + C1$$

So equation (i) become,

$$u = x^4/4 + y^4/4 + C_1$$

$$C_2 = x^4/4 + y^4/4 + C_1$$

$$C = x^4/4 + y^4/4$$

Answer.

Question # 3

(a)- $4y'' - 20y' + 25y = 0$

Solution:

The Auxilary equation is,

$$4\lambda^2 - 20\lambda + 25 = 0$$

$$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$\Rightarrow 2\lambda - 5 = 0 \quad \Rightarrow \quad \lambda_1 = 5/2$

$\Rightarrow 2\lambda - 5 = 0 \quad \Rightarrow \quad \lambda_2 = 5/2$

As, $\lambda_1 = \lambda_2 = 5/2$

The roots are real and equal,

So,

$$y = (c_1 + c_2 x)\, e^{\lambda x} = (c_1 + c_2 x)\, e^{5/2\, x}$$

Answer.

Question # 3

(b)- $4y'' - 6y' - 7y = 0$

Solution:

$$4y'' - 6y' - 7y = 0$$

Auxilary equation is,

$$4\lambda^2 - 6\lambda - 7 = 0$$

So, by quadratic formula,

$a = 4$, $b = -6$, $c = -7$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-7)}}{2(4)}$$

$$\lambda = \frac{6 \pm \sqrt{36 + 112}}{8}$$

$$\lambda = \frac{6 \pm \sqrt{148}}{8}$$

Take 2 as a common,

$$\lambda = \frac{3 \pm \sqrt{37}}{4}$$

So,

$$\lambda_1 = \frac{3 + \sqrt{37}}{4}$$

$$\lambda_2 = \frac{3 - \sqrt{37}}{4}$$

Roots are real. So,

$$y = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$y = C_1 e^{\frac{3+\sqrt{37}}{4} t} + C_2 e^{\frac{3-\sqrt{37}}{4} t}$$

Answer.

Thank You.