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Subject

MOS 2

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Submitted To :

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Question No : 1

Given Data:.

The state of stress which are given as;

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

Required Data:

Determine the state of stress at point rotated at 15° in clockwise direction.

Solution:.

The given stresses are

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

When we rotate the element of beam at 15° in clockwise direction

then we find:.

σ_{x_1} , σ_{y_1} and $\tau_{x_1 y_1}$

Now first we find σ_{x_1} :

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

So

$$\sigma_{x_1} = \frac{-40 + 70}{2} + \left(\frac{-40 - 70}{2} \right) \cos 2(-15^\circ) + (-50) \sin 2(-15^\circ)$$

$$\sigma_{x_1} = -7.63 \text{ MPa}$$

Now we have to find σ_{y_1}

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_{y_1} = \frac{-40 + 70}{2} - \left(\frac{-40 - 70}{2} \right) \cos 2(-15^\circ) - (-50) \sin 2(-15^\circ)$$

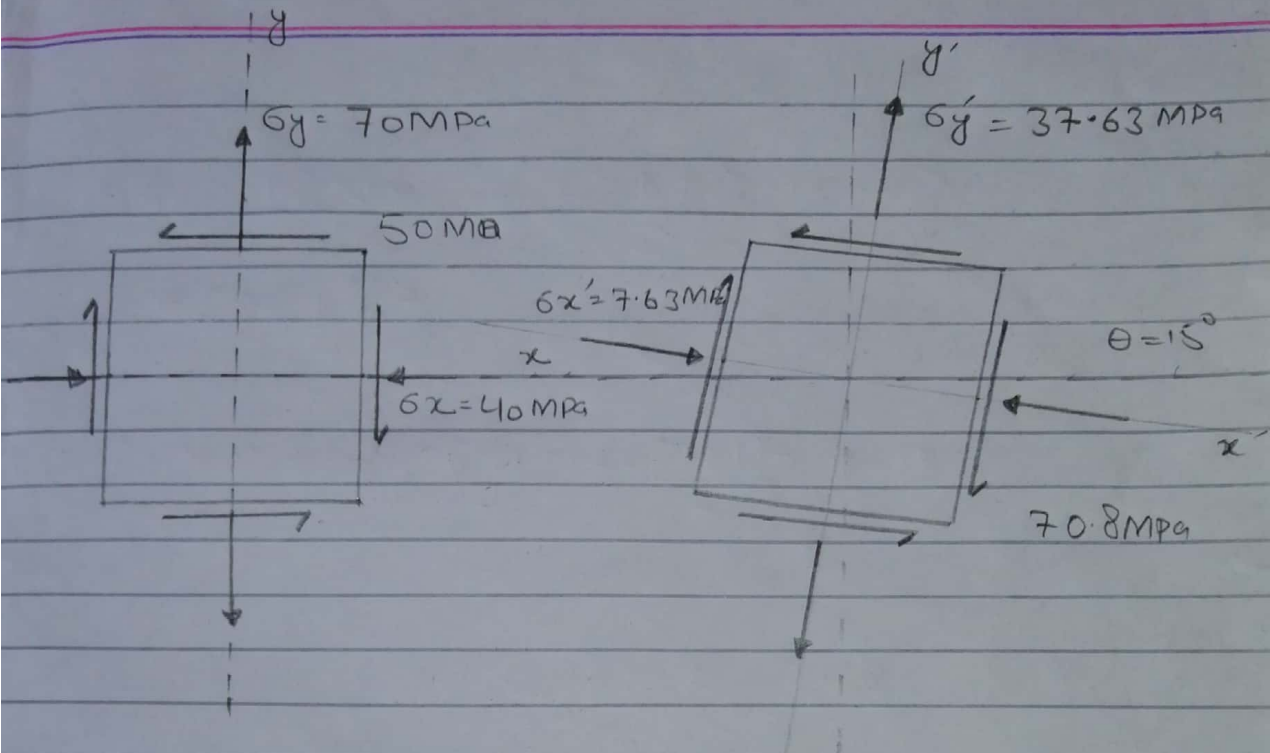
$$\sigma_{y_1} = 37.63 \text{ MPa}$$

Also we have to find $\tau_{x_1 y_1}$

$$\tau_{x_1 y_1} = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x_1 y_1} = -\left(\frac{-40 - 70}{2} \right) \sin 2(-15^\circ) + (-50) \cos 2(-15^\circ)$$

$$\tau_{x_1 y_1} = -70.80 \text{ MPa}$$



Q No # 2

Given Data:

The state of stresses which are given

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

a) First we find principle plane:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2(-50)}{-40 - 70} = \frac{2(-50)}{-40 - 70}$$

$$\tan 2\theta_p = 0.9091$$

$$2\theta_p = \tan^{-1}(0.9091)$$

$\theta_{p1} :$

$$\theta_p = \frac{42.27}{2}$$

$$\theta_{p1} = 21.14^\circ \text{ (for x-axis)}$$

$\theta_{p2} :$ $2\theta_p = 42.27 + 180$

$$\theta_p = \frac{222.27}{2}$$

$$\theta_{p2} = 111.735^\circ \text{ (for y-axis)}$$

Now we find principle stress as:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-40 + 70}{2} + \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$\sigma_1 = 89.33 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_2 = \frac{-40 + 70}{2} - \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$\sigma_2 = -59.33 \text{ MPa}$$

Now we gonna check in which angle goes with which principle stress;

$$\begin{aligned}\sigma_{x_1} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-40 + 70}{2} + \left(\frac{-40 - 70}{2}\right) \cos 2(21.4) + (-50) \sin 2(21.4)\end{aligned}$$

$$\sigma_{x_1} = -59.33 \text{ MPa}$$

Hence ;

$$\sigma_1 = 89.33 \text{ MPa with } \theta_{p_1} = 111.135^\circ$$

$$\sigma_2 = -59.33 \text{ MPa with } \theta_{p_2} = 21.14^\circ$$

Part B

Given Data

$$\sigma_x = -40$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

Required Data

Max In-plane Shear Stress

$$\tau_{\max} \text{ in plane} = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$= \sqrt{\frac{(-40 - 70)^2}{4} + (-50)^2}$$

$$= 74.3 \text{ MPa}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{-40 + 70}{2}$$

$$\sigma_{\text{avg}} = 15$$

Now we have find Max Shear plane

$$\tan 2\theta_s = \frac{(\sigma_x - \sigma_y) / 2}{\tau_{xy}}$$

$$= \frac{(-40 - 70) / 2}{\tau_{xy}}$$

$$= \frac{(-40 - 70) / 2}{-50}$$

$$\tan 2\theta_s = -1.1$$

$$\theta_s = \frac{1}{2} \tan^{-1} (-1.1)$$

$$\theta_s = -23.9^\circ$$