NAME # KIRAMAT ULLAH

ID # 13290

SUBJECT # COMPLEX AND MULTI VARIABLE CALCULUS

DEPARTMENT # BEE

DATE # 21-08-2020

TEACHER NAME

ENGR MUJTABA IHSAN

QUESTION

NO 1

PART A

Express $-3+4i$ in polar form and represent it graphically.

SOLUTION:

$$z = -3+4i$$

$$z = r(\cos\theta + i\sin\theta)$$

$$r = \sqrt{(x^2+y^2)}$$

$$r = \sqrt{(-3)^2+(4)^2} = \sqrt{9+16}$$

$$r = \sqrt{25} = 5$$

$$\theta = \tan^{-1}(y/x)$$

$$\theta = \tan^{-1}(-4/3) = \tan^{-1}(-1.33)$$

$$\theta = -53.13$$

$$z = r(\cos\theta + i\sin\theta)$$

$$= 5(\cos -53.13 + i\sin -53.13)$$

$$z = 5\cos -53.13 + i5\cos\sin -53.13$$

53·13

QUESTION

NO 1

PART B

b) Given that $u(x,y) = (x^3+3xy^2) + i(3x^2y - y^3)$.
Determine if the function is analytic or not?

SOLUTION:

Since $u(x,y) = x^3 + 3xy^2$ and
$v(x,y) = 3x^2y - y^3$ we have

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 , \quad \frac{\partial v}{\partial y} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial v} = -6xy , \quad \frac{\partial v}{\partial x} = 6xy$$

Hence the couchy-Rieman equation are satisfied and $f(z)$ is analytic the complex derivative of

$f(z)$ is $f'(z) = 3x^2 - 3y^2 + i(6xy)$

$$\boxed{= 3z^2}$$ Answer.

QUESTION NO 2

If $z_1 = 5+3i$ and $z_2 = 4-2i$, evaluate $z_1 z_2$ and z_1/z_2.

SOLUTION:

$$z_1 z_2 = (5+3i)(4-2i)$$

$$z_1 = 5+3i$$
$$z_2 = 4-2i$$

$$z_1 z_2 = 20 - 10i + 12i - 6i^2$$

$$= 20 + 2i - 6(-1)$$

$$= 20 + 2i + 6$$

$$= \boxed{2i + 26}$$ Answer.

$$\Rightarrow \frac{z_1}{z_2} = \frac{5+3i}{4-2i}$$

$$= \frac{5+3i}{4-2i} \times \frac{4+2i}{4+2i}$$

$$= \frac{20+10i+12i+6i^2}{16+4}$$

$$= \frac{20+22i+6(-1)}{20}$$

$$= \frac{20-6+22i}{20}$$

$$= \frac{14+22i}{20}$$

$$= \boxed{\frac{14}{20} + \frac{22}{20} i}$$ Answer

QUESTION NO 3

Given that $u(x,y) = x^3 - 3xy^2 - 5y$. Determine if the function is ~~analytic~~ harmonic? if so, evaluate the conjugate harmonic function of u.

SOLUTION :-

$\partial u / \partial x = 3x^2 - 3y^2$

again differentiation

$$\frac{\partial^2 u}{\partial x^2} = 6x$$

$$\frac{\partial u}{\partial y} = -6xy - 5$$

again differentiate

$$\frac{\partial^2 u}{\partial y^2} = -6x$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6x = 0$$

Now

$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 3x^2 - 3y^2$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} =$

$6xy + 5$

Integrating the first one

$v(x,y) = 3x^2y - y^3 + h(x)$

and $\frac{\partial v}{\partial x} = 6xy + h'(x)$, $h'(x) = 5$, $h(x) =$

$5x + C$

Thus $v(x,y) = 3x^2y - y^3 + 5x + C$ Ans.

QUESTION NO 4

PART A

$f(z) = z^2 / 5z+2$

SOLUTION:

$$= \frac{5z+2 \, d/dz \, z^2 - z^2 \, d/dz \, 5z+2}{(5z+2)^2}$$

$$= \frac{(5z+2)(2z) - z^2(5)}{(5z+2)^2}$$

$$= \frac{10z^2 + 4z - 5z^2}{(5z+2)^2}$$

$$= 10z^2 - 5z^2 + 4z / (5z+2)^2$$

$$\boxed{= 5z^2 + 4z / (5z+2)^2}$$

Answer

QUESTION NO 4

PART B

Differentiate the following:

ii) $f(z) = 3z^4 - 5z^3 + 2z + 1$

SOLUTION:

$f(z) = 3z^4 + 5z^3 + 2z + 1$

take derivative

$f'(z) = \frac{d}{dz} 3z^4 + \frac{d}{dz} 5z^3 + \frac{d}{dz} 2z + \frac{d}{dz} 1$

$f'(z) = 12z^3 + 15z^2 + 2$ Ans.