Date: ID: 15404 ①

Q.1.a

Flexibility Method Procedure:

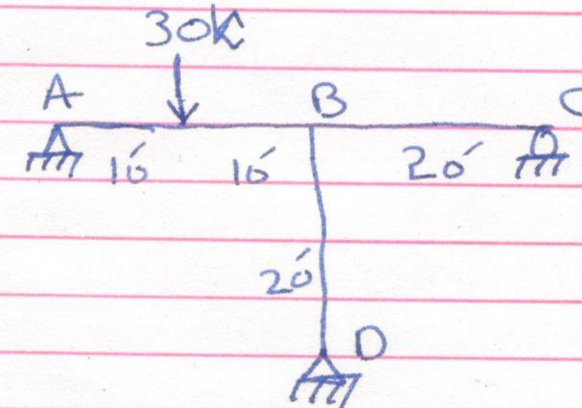
In flexibility method, static indeterminancy of the structure is calculated.
Then redundant members or supports (equal to ~~num~~ static indeterminancy) are removed until structure becomes statically determinate.
The displacement of some points in the released structure is then determined by, say, the unit load method.
The actual loads on the structure are removed and unknown forces applied to the points where the structure has been released;
The displacement at the point produced by these unknown forces must, from compatibility, be the same as that in the released structure.
The unknown forces are then obtained.
This approach is known as the flexibility method.

Q.1 (b).

Flexibility Method	Stiffness Method
(1) Convert the indeterminate structure to a determinate one by removing some unknown forces/support reactions and replacing them with (assumed) known/unit forces.	Express local (member) force-displacement relationships in terms of unknown member displacements.
2). Using superposition, calculate the force that would be required to achieve compatibility with the original structure.	Using equilibrium of assembled members, find unknown displacements.
3) Unknowns to be solved for are usually redundant forces.	Unknowns are usually displacements
4) Coefficients of the unknowns in equations to be solved are 'flexibility coefficients.	Coefficients of unknowns are stiffness coefficients.

Q.No (02)

a): Analyze Frame using Flexibility method.

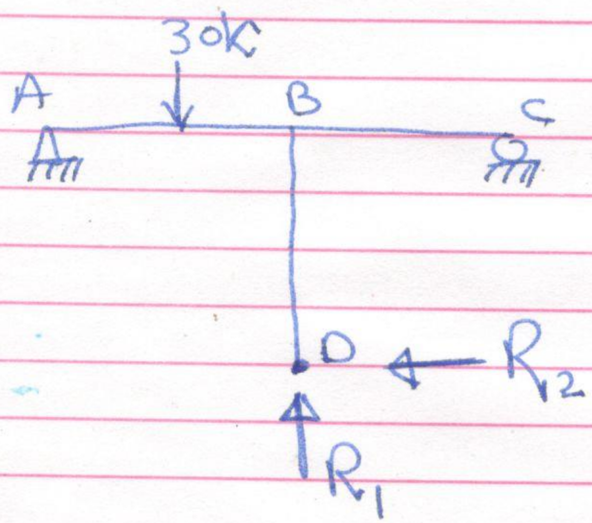


EI = Constant

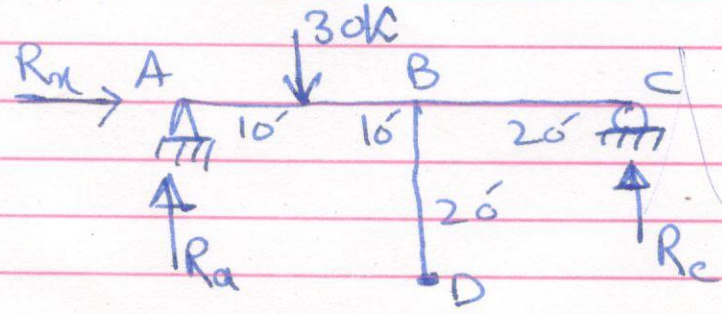
Static Indeterminacy = S.I = 3m + r − 3j

$= 3\times3 + 5 - 3\times4 = 2$

Taking vertical and Horizontal support reaction at D as redundant (R_x & R_y) or (R_1 & R_2).



Calculating M values under external loads on Free structure i.e



ΣM at $c = 0 \Rightarrow R_a \times 40 - 30 \times 30 = 0$

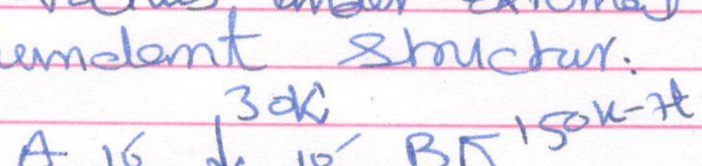
$\Rightarrow R_a = 22.5K$

$\Sigma F_y = 0 \Rightarrow 22.5 - 30 + R_c = 0$

$\Rightarrow R_c = 7.5K$

$\Sigma F_x = 0 \Rightarrow R_x = 0.$

M-values under external loading in Redundant structur.



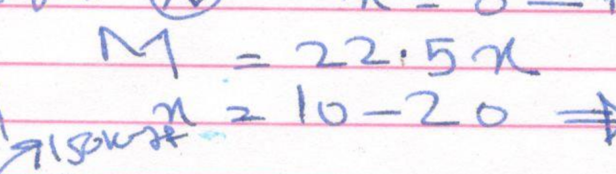
Member AB:

origin-A $x = 0 - 10,$

$M = 22.5x$

& $x = 10 - 20 \Rightarrow M = 22.5x - 30(x-10)$

$= 22.5x - 30x + 300$



BC:

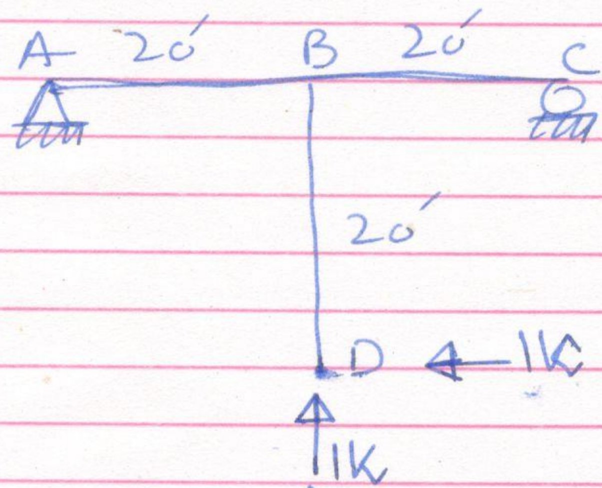
origin C: $x = 0 - 20$

$M = 7.5x$

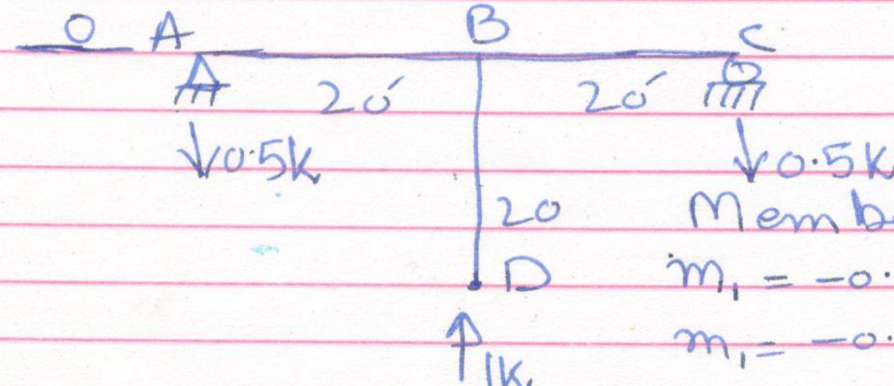
Member BD:

$M = 0$

Now applying unit actions in direction of redundant reactions and computing corresponding displacements under unit loads. i.e



Now computing m_1 values corresponding unit load (1) i.e



Member AB:

$m_1 = -0.5x \quad (x = 0 - 10)$

$m_1 = -0.5x \quad (x = 10 - 20)$

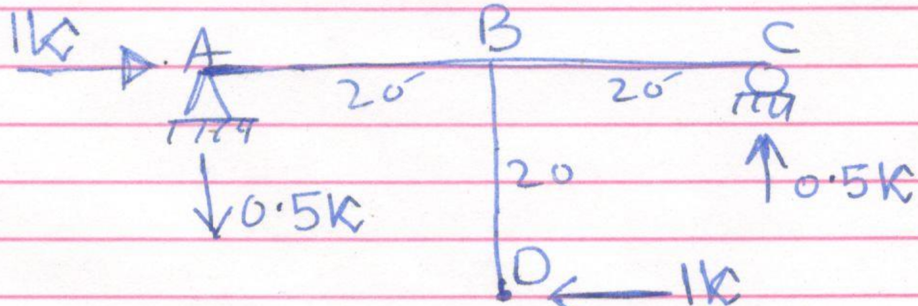
Member BC:

$$m_1 = -0.5x \quad (x = 0-20').$$

Member BD: origin (D)

$$m_1 = 0$$

Now m_2 values under 2nd unit force i.e



Member AB: origin (A)

$$m_2 = -0.5x \quad (x = 0-10')$$

$$m_2 = -0.5x \quad (x = 10-20').$$

Member BC: origin (C)

$$m_2 = +0.5x \quad (0-20')$$

Member BD: origin (D)

$$m_2 = +1 \cdot x \quad (x = 0-20')$$

Member	AB	AB	BC	BD
origin	A		C	D
Limit	0–10	10–20	0–20	0–20
M	$22.5x$	$300-7.5x$	$7.5x$	0
m_1	$-0.5x$	$-0.5x$	$-0.5x$	0
m_2	$-0.5x$	$-0.5x$	$0.5x$	$1.x$

Now vertical displacement in redundant structure is

$$\Delta_{LV} = \frac{1}{EI}\int_1^m (M)(m_1)\,dx$$

$$= \frac{1}{EI}\left[\int_0^{10}(22.5x)(-0.5x)\,dx + \int_{10}^{20}(300-7.5x)(-0.5x)\,dx + \int_0^{20}(7.5x)(-0.5x)\,dx + 0\right]$$

$$= \frac{1}{EI}\left[(-11.25)\left|\frac{x^3}{3}\right|_0^{10} + (-150)\left|\frac{x^2}{2}\right|_{10}^{20} + 3.75\left|\frac{x^3}{3}\right|_{10}^{20} + (-3.75)\left|\frac{x^3}{3}\right|_0^{20}\right]$$

$$= \frac{1}{EI}\left[-3750 - 22500 + 8750 - 10000\right]$$

$$\Delta_{LV} = -27500/_{EI}.$$

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Now Horizontal displacement in redundant structure is

$$\Delta_{LH} = \frac{1}{EI}\left[\int_1^m (M)(m_2)\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^{10}(22.5x)(-0.5x)\,dx + \int_{10}^{20}(300-7.5x)(-0.5x)\,dx\right.$$

$$\left. + \int_0^{20}(7.5x)(0.5x)\,dx + 0\right]$$

$$= \frac{1}{EI}\left[-3750 - 22500 + 8750 + 10{,}000\right]$$

$$\Delta_{LH} = -7500/EI$$

Now Calculating displacements corresponding to application of unit forces i.e Flexibility coefficients

$$F_{11} = \frac{1}{EI}\int_{i=1}^{m}(m_1)(m_1)\,dx = \frac{1}{EI}\left[\int_0^{10}(-0.5x)^2dx + \int_{10}^{20}(-0.5x)^2dx\right.$$

$$\left. + \int_0^{20}(-0.5x)^2dx + 0\right]$$

$$= 1333.33/EI$$

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$$F_{12} = F_{21} = \sum_{i=1}^{m} \int_0^L \frac{(m_1)(m_2)}{EI} dx$$

$$= \frac{1}{EI} \left[\int_0^{10} (-0.5x)(-0.5x)dx + \int_{10}^{20} (-0.5x)(-0.5x)dx \right.$$

$$\left. + \int_0^{20} (-0.5x)(0.5x)dx + 0 \right]$$

$$= \frac{1}{EI} \left[\frac{0.25}{3}(10)^3 + \frac{0.25}{3}\left[(20)^3 - (10)^3\right] \right.$$

$$\left. - \frac{0.25}{3}(20)^3 \right]$$

$$= 0$$

$$F_{22} = \sum_{i=1}^{m} \int_0^l \frac{(m_2)(m_2)}{EI} dx$$

$$= \frac{1}{EI} \left[\int_0^{10} (-0.5x)^2 dx + \int_{10}^{20} (-0.5x)^2 dx + \int_0^{20} (0.5x)^2 dx \right.$$

$$\left. + \int_0^{20} (x)^2 dx \right] = 4000/EI$$

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Now, Vertical and Horizontal displacements at 'D' is zero, so

Compatibility Equation can be written as;

$$\Delta V_{E} = \Delta LV + f_{11} R_1 + f_{12} R_2 = 0$$

$$\Delta H = \Delta LH + f_{21} R_1 + f_{22} R_2 = 0$$

Putting values in above Equations and solving for R_1 & R_2

$$-\frac{27500}{EI} + \frac{1333.33}{EI} R_1 + 0 \times R_2 = 0$$

$$\Rightarrow \quad R_1 = +20.625K \uparrow \text{(upward)}$$

$$-7500/EI + \frac{4000}{EI} * R_2 + 0 \times R_1 = 0$$

$$\Rightarrow \quad R_2 = +1.875K \ (\leftarrow)$$

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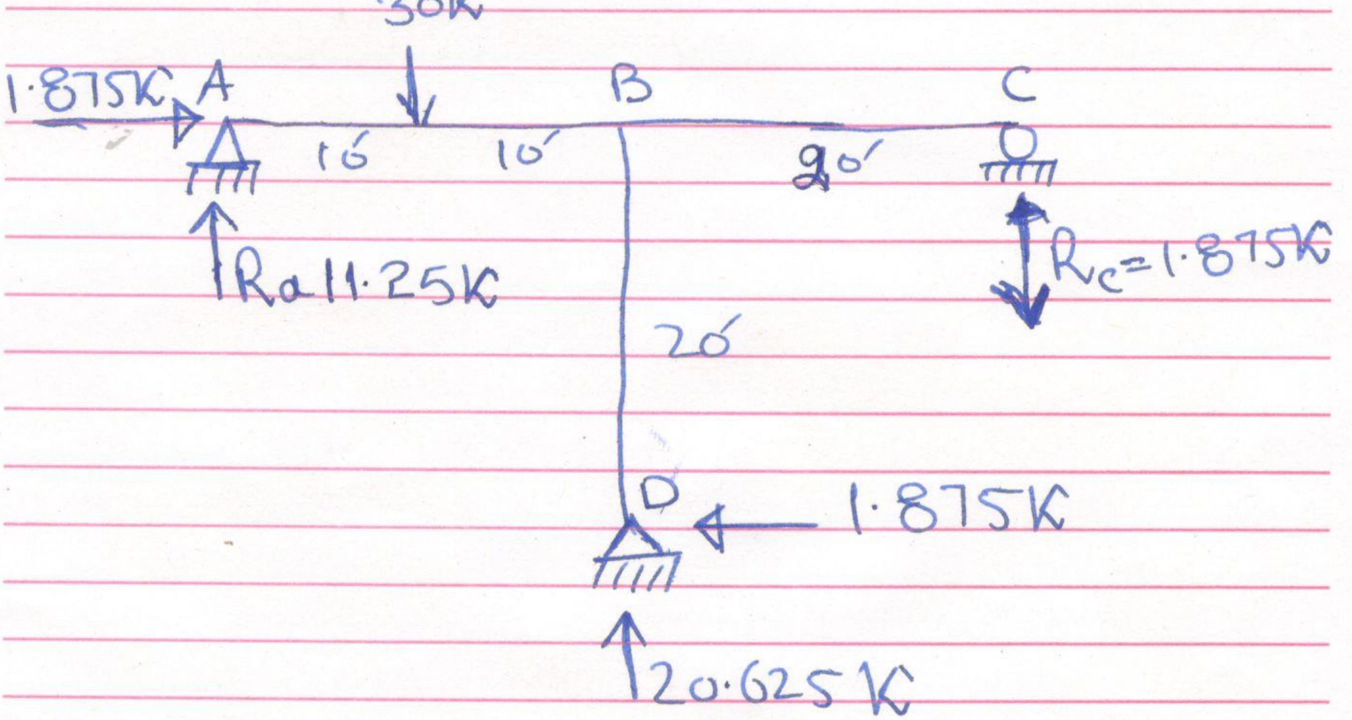


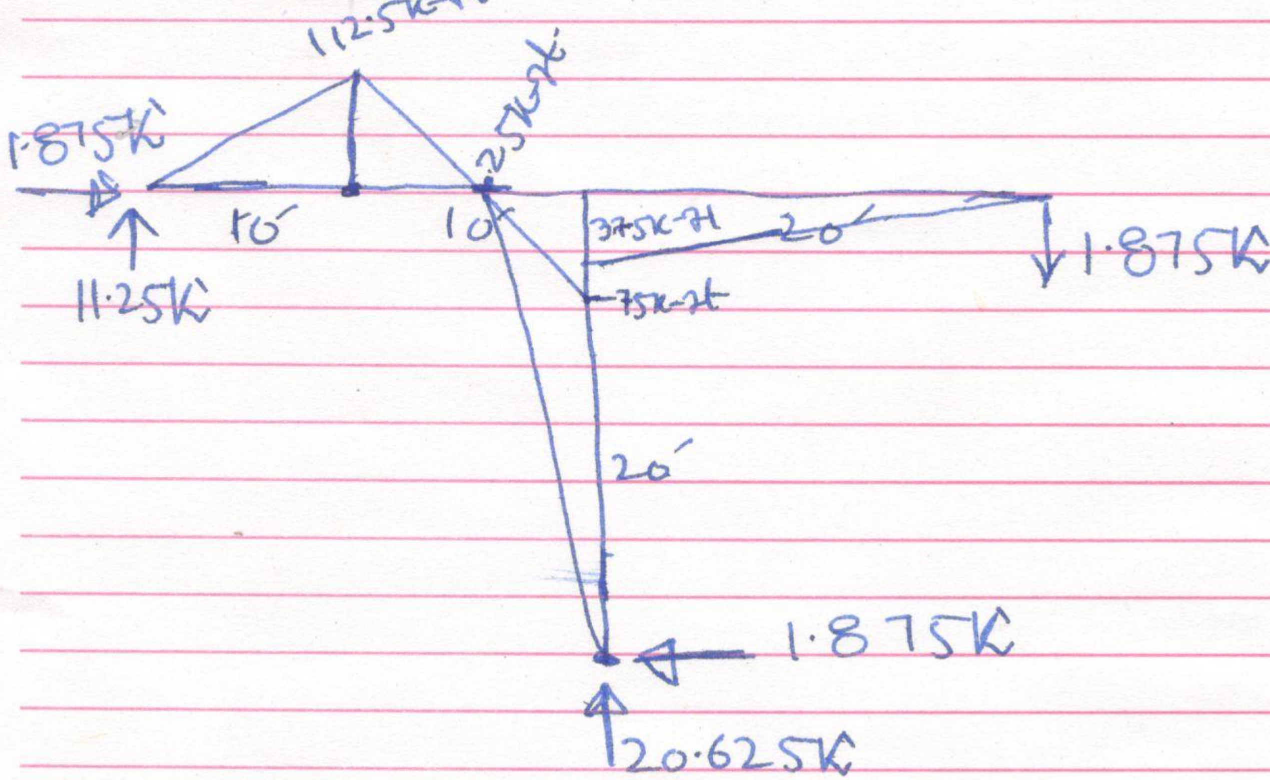
ΣM at (C) $=0 \Rightarrow$

$R_a \times 40 - 30 \times 30 + 20.625 \times 20 + 1.875 \times 20 = 0$

$\Rightarrow \quad R_a = +11.25K\ (\uparrow)$.

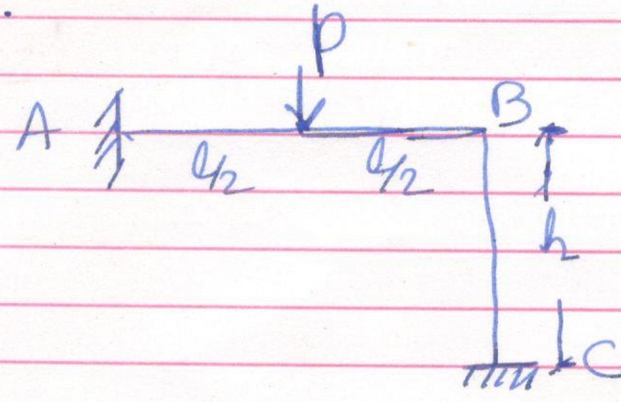
Now $\quad \Sigma F_y = 0 \Rightarrow 11.25 - 30 + 20.625 + R_c = 0$

$\Rightarrow \quad R_c = -1.875K$ ($\downarrow$ downward).



Hence, the required Solution is above.

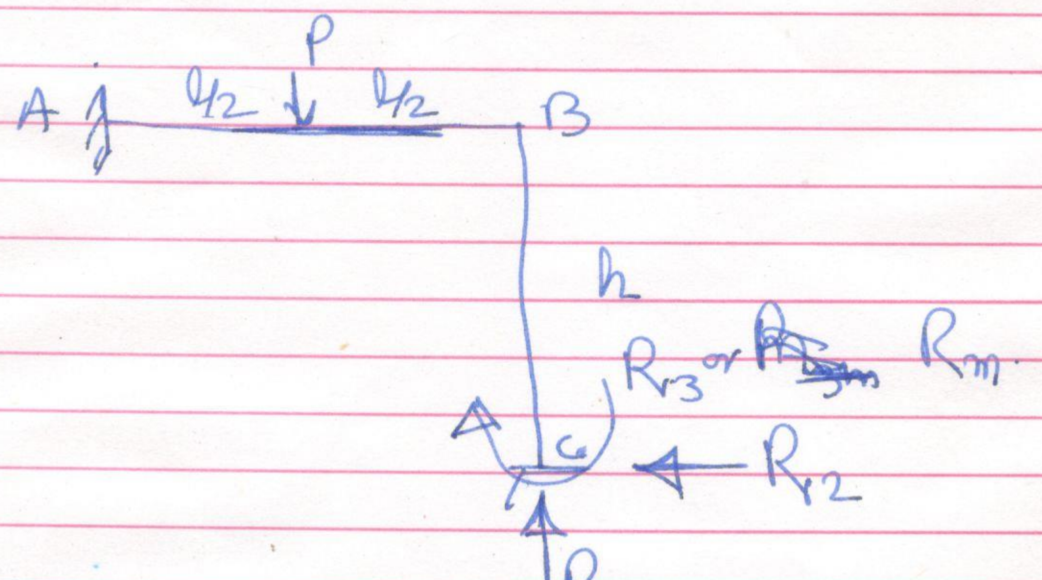
Q:2 (b):



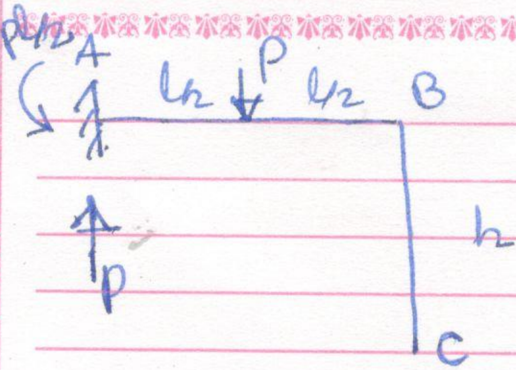
Soln:

$$S.I = 3m + r - 3j$$
$$= 3\times2 + 6 - 3(3) = 3^{\circ}$$

Taking three reactions at C redundants i.e

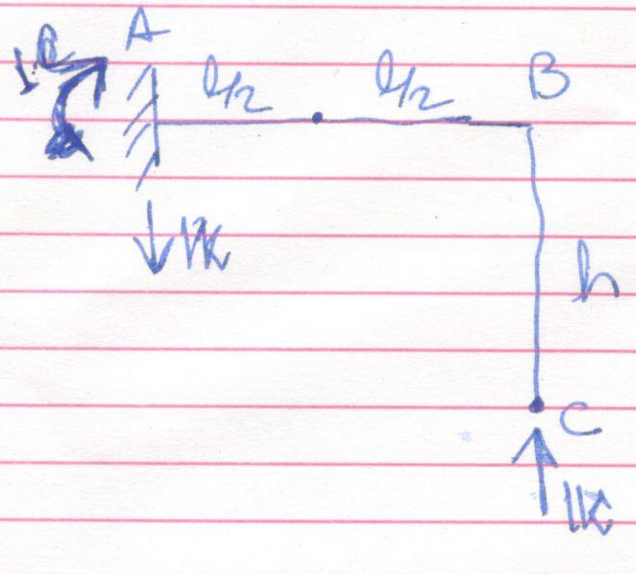


Now 'M' values on redundant (free) structure under external loading i.e



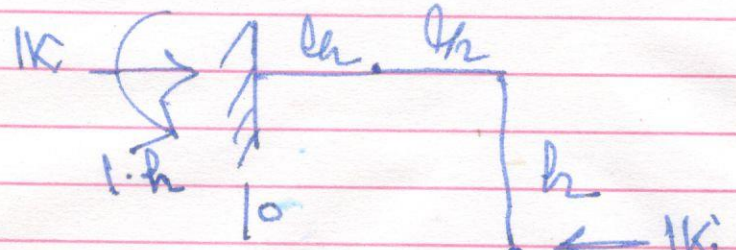
(A)

Now, Redundant structure under unit load corresponding to redundant reaction R_1 i.e



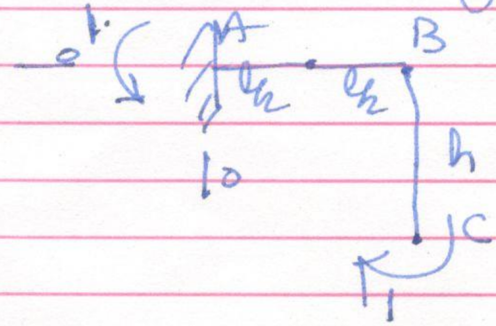
(B)

Now, unit load corresponding to R_2 i.e



(C)

Now unit action corresponding to R_m i.e



Now, tabulating values of moment w.r.to length of the structure for calculating corresponding displacements i.e

	Member	AB		BC
	Origin	A		C
	limit	$0 - l/2$	$l/2 - l$	
Fig (A)	M	$Px - Pl/2$	$Px - Pl/2 - P(x - l/2) = 0$	0
Fig (B)	m_1	$l - x$	$l - x =$	0
Fig (C)	m_2	$-1 \cdot h$	$-h$	$+h$
Fig (D)	m_3	-1	-1	$+1$

Now displacement in ~~redundant~~ structure corresponding to redundant Reaction R_1 (under external loading i.e

$$\Delta_1 = \sum_{i=1}^{m} \left[\int_0^l \frac{(M_x)(m_{1x})\, dx}{EI} \right]$$

Date:

$$\Rightarrow \Delta_{1L} = \frac{1}{EI}\left[\int_0^{l/2} (Px - Pl/2)(l-x)\,dx + 0 + 0\right]$$

$$\Delta_{1L} = \left(\frac{3}{8}Pl^3 - \frac{1}{3}Pl^2\right)\frac{1}{EI}$$

Now displacement in structure corresponding to redundant Reaction R_2 under external loading i.e

$$\Delta_{2L} = \frac{1}{EI}\sum_{i=1}^{m}\int_0^{l}(M)(m_2)\,dx$$

$$\Rightarrow = \frac{1}{EI}\left[\int_0^{l/2}(Px - Pl/2)(-h)\,dx + 0 + 0\right]$$

$$\Delta_{2L} = 0$$

Now displacements Δ_{3L} i.e

$$\Delta_{3L} = \frac{1}{EI}\sum_{i=1}^{m}\int_0^{l}(M)(m_3)\,dx$$

$$\Delta_{3L} = 0$$

Now Calculating flexibility coefficients

$$F_{11} = \frac{1}{EI}\sum_{i=1}^{m}\int_0^{l}(m_1)^2\,dx$$

Date:

$$= \frac{1}{EI}\left[\int_0^{l/2}(l-x)^2\,dx + \int_{l/2}^{l}(l-x)^2\,dx + 0\right]$$

$$F_{11} = \frac{1}{EI}\left(\frac{3}{4}l^3 - l^2\right)$$

$$F_{22} = \frac{1}{EI}\sum_{i=1}^{m}\int_0^{l}(m_2)^2\,dx$$

$$= \frac{1}{EI}\left[\int_0^{l/2}h^2\,dx + \int_{l/2}^{l}h^2\,dx + \int_0^{h}h^2\,dh\right.$$

$$F_{22} = \left(h^2 l + h^3/3\right)\frac{1}{EI}$$

$$F_{33} = \cancel{\frac{1}{EI}}\sum_{i=1}^{m}\int_0^{l}\frac{(m_3)^2\,dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{l/2}dx + \int_{l/2}^{l}dx + \int_0^{h}dh\right]$$

$$F_{33} = (l+h)/EI$$

$$F_{12} = F_{21} = \sum_{i=1}^{m}\int_0^{l}\frac{(m_1)(m_2)\,dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{l/2}(l-x)(-h)\,dx + \int_{l/2}^{l}(l-x)(-h)\,dx + 0\right]$$

$\cancel{+0}$

$$F_{12} = F_{21} = -hl^2/2EI$$

$$F_{13} = F_{31} = \sum_{i=1}^{m}\int_0^l \frac{(m_1)(m_3)\,dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{l/2}(l-x)(-1)\,dx + \int_{l/2}^{l}(l-x)(-1)\,dx + 0\right.$$

$$F_{13} = F_{3} = -\frac{3}{8}l^2$$

$$F_{23} = F_{32} = \sum_{i=1}^{m}\int_0^l \frac{(m_2)(m_3)\,dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{l/2}(h)\,dx + \int_{l/2}^{l} h\,dx + \int_0^h h\,dh\right].$$

$$F_{23} = F_{32} = hl + h^2/2$$

Now writing compatibility equations for three displacements i.e vertical, horizontal and rotation at (C) i.e

$$\Delta_V = \Delta_{1L} + f_{11}R_1 + f_{12}R_2 + f_{13}R_{3m} = 0 \quad (1)$$

$$\Delta_H = \Delta_{2L} + f_{21}R_1 + f_{22}R_2 + f_{23}R_m = 0 \quad (2)$$

&

$$\theta = \Delta_{3m} + f_{31}R_1 + f_{32}R_2 + f_{33}R_3 = 0 \quad (3)$$

where

Δ_{1L}, Δ_{2L} & Δ_{3m} = displacements in released structure due to external loading.

$f_{11}, f_{12} \ldots f_{33}$ = Flexibility coefficients corresponding unit action against each displacement

&

R_1, R_2 & R_m = Redundant Reactions (unknowns).

Putting calculated values in above three equations and solving for unknowns, we get

$$R_2 = \frac{8/3P - 3Pl}{1 - 6l + 5h} \quad (\text{Kips})$$

$$R_1 = \left(\frac{2h}{3l} + \frac{5h}{3l^2}\right) * R_2 \quad (\text{Kips})$$

&

$$R_3 = -h/3 * R_2 \quad (\text{k-ft}).$$

The remaining three unknowns in the structure can be obtained by three equations of Equilibrium.
