Assignment - 1 ~~Discrete~~ Mathe
Name = Muhammad Amir Algebra
~~ID~~: ID = 16372 Page-1

Question No 1:
Define the following terms.

Q 1:
Orthogonal matrix.
A Square matrix with real number or elements is said to be an orthogon matrix, if its transpose is equal to its inverse matrix.

2: Basis for a vector Space.

a Set B of elements vectors in a vector Space v is Called a basis, if every element of v may be written in a unique way as a (finite) linear Combination of elements of B

3: Span Set of vector Space.
the linear span (also Called the linear hull or ~~Just~~ Just span) of a set S of vectors in a vector Space is the Smallest linear Subspace that Contains the Set ... the linear span of a set of vectors is therefore a vector Space.
4: The dimension of a vector Space v is the Cardinality of a basis of v over its base field.

5:
~~Eing~~ Eigen vector.

an eigenvector or Characteristic vector of a linear transformation is a nonzero vector that Changes at most by a Scalar factor when that linear transformation is applied to it.
The Corresponding eigenvalue is the factor by which the eigenvector is Called.

6: vector Subspace.

it is avector Space that is a Subset of Some larger vector Space. A linear SubSpaceis usually Called Simply
a Subspace when the Context Serves to distinguish it from other types of Subspace.

7: Kernel of a linear transformation.

The Kernel of a linear transformation L is the Set of all vectors v Such that.
L (v) = 0.

8: Nullity of a linear transformation.

the nullity of a linear transformation of vector spaces is the dimension of its null space.

9: Image of a linear transformation.

The image of a linear transformation matrix is the span of the vectors of the linear transformation.

10: Rank of linear transformation.

The rank of a linear transformation L is the dimension of its image. written rankL.

11: Characteristic polynomial of a square matrix.

The Characteristic Polynomial of a square matrix is a polynomial which is invariant under matrix similarity and has the eigenvalues as roots. it has the determinant and the trace of the matrix as coefficients

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12 Equavilance relation.

a relation between elements of Set which is reflexive, Symmetric, and which defines exclesive Classes whose members bear the relation other and not to those in other Classes.

13 A homogenous Solution to a linear System of equations.

For a homogenous System of equations $ax+by=0$ and $cx+dy=0$ the situation is Slighty different....
A nxn.
homogenous System of linear equations has a unique Solution (the trivial Solution) if and only if its determinant is non-zero. if this determinant is zero, then the system has an infinite number of solution.

14 A Particular Solution to a linear System equations.

For a System involving two Varible (x and y), each linear equation determines a Solution to a linear system must satisfy all of the intersection of these lines. and it is hence either a line, a single point, or the empty set.

15 The general solution to a linear system of equations:

The sum of two such pairs and multiplication of a pair with a number is defined as follows;

$(x_1, y_1) + (x_2, y_2) =$

$(x_1, x_2, y_1 + y_2)$ a $(x, y) = (ax, ay)$.

16 The direct sum of a pair of subspace of a vector space.

The direct sum of a pairs of subspace of a vector space multiplication sum of two such.

Question No. 2:

Consider the system of equations.

$$x+y+z+w=1$$
$$x+2y+2z+3w=1$$
$$x+2y+3z+3w=1$$

Answer No. 3:

$$x+y+z+w=1$$
$$x+2y+2z+3w=1$$
$$x+2y+3z+3w=1$$

Express the matrix as $Mx=V$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{vmatrix} = \begin{vmatrix} x \\ y \\ z \\ w \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x \\ y \\ z \\ w \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 0 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ w \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x \\ y \\ z \\ w \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$$

$$\Rightarrow 3z+3w=0$$

$$\boxed{= \quad z=-w}$$ The solution is trivial.

Question No: 3:

Compute the following determinants.

$$\det \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

Answer No. 3:

Solution:

$$(1\times4)-(2\times3)$$
$$= 4-6=-2$$

$$\det \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$= (45-48) - [2(36-42)] + 3[(32)-(35)$$

$$= -3 - (-12) + 3(-2)$$
$$= -3+12-9$$
$$= 0$$

Q4: In the limit of small activator diffusivity, the stability of symmetric K-spike equilibrium solution to the Gierer-Meinhardt reaction - diffusion system is a one - dimensional spatial domain is studied for various ranges of the reaction - time constant $\tau \geq 0$ and the diffusivity $D > 0$ of the inhibitor field dynamics. A nonlocal eigenvalue problem is derived that determines the stablity on an $O(1)$ time-scale of these K-slipe equilibrium patterns. The spectrum of this eigenvalue problem is studied in detail using a combination of rigorous, asymptotic and numeric method.

For $K = 1$ and for various exponent sets of the nonlinear terms, we show that for each $D > 0$, a one-spike solution is stable only when $0 \leq \tau < \tau_0(D)$. As τ increase past $\tau_0(D)$, a pair of complex conjuedge-