SHAH HASSAN.

ID- 7978.

SEC- B.

Sub- Numerical Analysis.

To,

Ma'm Shumaila Mazhar.

Q No-01

Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Use Newton raphson method with

$$x_0 = 0.02$$

Solution:

Rearranging of the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

$$\boxed{x_1 = 0.081}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.081 - \frac{(-0.000016)}{-0.0070}$$

$f(x_1) = -0.000016$

$f'(x_1) = -0.0070$

$$\boxed{x_2 = 0.058}$$

$$\Rightarrow \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$f(x_2) = 0.000030$

$f'(x_2) = 0.0090$

$$\boxed{x_3 = 0.061}$$

Q. No-02

Use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the lagrange interpolation polynomial for $f(x) = \frac{1}{x}$ at $x = 3$

Solution:

Putting the values in the funtion.

$$y_0 = 0.5, \quad y_1 = 0.364, \quad y_2 = 0.25$$

$$y = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y^0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1' + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2^3$$

$$= \frac{(3 - 2.75)(3 - 4)}{(2 - 2.75)(2 - 4)}(0.5) + \frac{(3 - 2)(3 - 4)}{(2.75 - 2)(2.75 - 4)}(0.364)$$

$$+ \frac{(3 - 2)(3 - 2.75)}{(4 - 2)(4 - 2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\Rightarrow \boxed{y = 0.33} \text{ Ans.}$$

Q No-03

Complete the divided difference table for the given and construct the interpolating polynomial that uses all this data

$x = 1.0 \quad 1.3 \quad 1.6 \quad 1.9 \quad 2.2$

$Y = 0.7651977 \quad 0.6200860 \quad 0.4554022 \quad 0.2818186 \quad 0.1103623$

	x_i	$f(x_i)$	$f(x_{i-1})(x_i)$	$f(x_{i-2})(x_{i-1})(x_i)$	$f(x_{i-3})(x_i)$	$f(x_{i-4}\cdots,x_i)$
x_0	1	0.7651977				
			-0.4837056			
x_1	1.3	0.6200860		-0.108734		
			-0.548946		0.0658785	
x_2	1.6	0.4554022		-0.0494433		-0.0028049
			-0.578612		0.06251255	
x_3	1.9	0.2818186		0.006818		
			-0.571521			
x_4	2.2	0.1103623				

(i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$

(ii) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$

(iii) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

(iv) $$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$$

$$\boxed{f(x_3, x_4) = -0.571521}$$

Second divided difference;

$$f(x_0, x_1, x_3) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$\boxed{= -0.108734}$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$\boxed{f(x_1, x_2, x_3) = -0.0494433}$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$\boxed{f(x_2, x_3, x_4) = 0.006818}$$

Third divided difference;

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (0.108734)}{1.9 - 1}$$

$$\boxed{f(x_0, x_1, x_2, x_3) = 0.0658785}$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$f(x_1, x_2, x_3, x_4) = 0.06251255$$

Forth divided diffevence

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$f[x_0, x_1, x_2, x_3, x_4] = 0.0028049$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3)(-0.108734) + (x-1)(x-1.3)(x-1.6)(0.0658785) + (x-1)(x-1.3)(x-1.6)(x-1.9)(-0.0028049).$$