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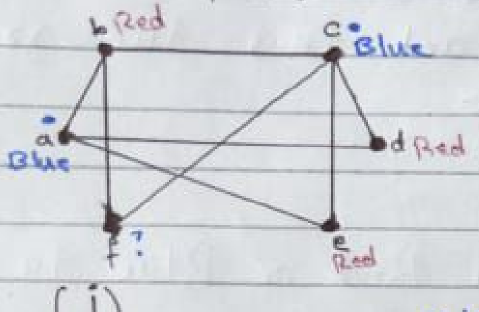
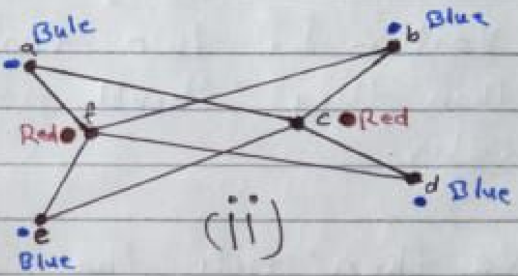
Subject = Discrete structure

Final assigment spring 2020.

Q1. Determine wether the graph are bipartite:-




(i)




(ii)

Ans ) i) Solution:- Let us assign red or blue to each vertex.
If two vertices are connected. Then they should not have the same color.

- We start by assigning "Blue" to a. Since b is connected to the "Blue" a. We assign "Red" to b.
Since C is connected to the "Red" b. We assign "Blue" to C.
We then not that f is connected to the Red b and the blue to c. which means that we can not assign a color to f such that it differs from the color of the connected vertices. As are only two colors: Red and Blue.

②

Thus it is not possible to assign Red or Blue to each vertex such that connected vertices do not have the same color and thus the graph is not bipartite.

(ii) Ans) Lets us assign Red or Blue to each vertex. If two vertices are connected then they should not have the same color.

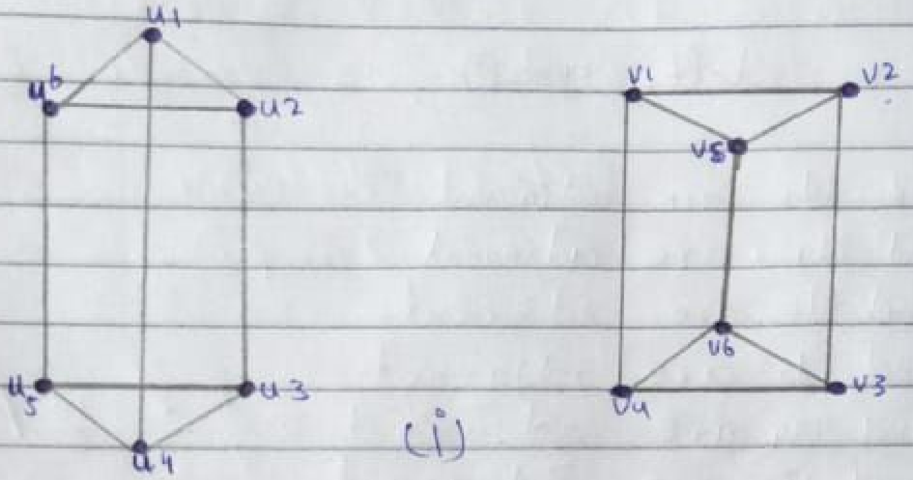
We then note that it is possible to assign Red or Blue to each vertex such that connected vertices do not have the same color and thus the graph is bipartite.

Moreover the partit joning of the vertices are the set with the blue vertices and the set with red vertices

$V_1 = \{a, b, d, e\}$

$V_2 = \{c, f\}$

Q.2) Determine wether the given pair of graphs is isomorphic?




(i)

A) Lets first determine that set of vertices and set of edge of left graph

$V_1 = \{u_1, u_2, u_3, u_4, u_5\}$

$E_1 = \{(u_1, u_2), (u_6, u_4), (u_1, u_5), (u_2, u_3), (u_2, u_4), (u_2, u_5), (u_3, u_4), (u_4, u_5)\}$

let us first determine the set of vertices the set of edge of the right of graph

$V_2 = \{(u_1, u_5), (u_1, u_3), (u_1, u_4), (u_5, u_2)$
$(u_5, u_3), (u_5, u_4), (u_2, u_3), (u_3, u_4)$

By comparing two sets of edges. We can define the following one-to-one and onto function f from $V_1$ to $V_2$,

$f(u_1) = v_1$
$f(u_2) = v_5$
$f(u_3) = v_2$
$f(u_4) = v_3$
$f(u_5) = v_4$

f is then a function that makes the two graph isomorphic. Since

edge in left graph-

$U_1$ and $U_2$ are adjacent.
$U_1$ and $U_4$ are adjacent.
$U_1$ and $U_5$ are adjacent
$U_2$ and $U_3$ are adjacent
$U_2$ and $U_4$ are adjacent
$U_2$ and $U_5$ are adjacent
$U_3$ and $U_4$ are adjacent
$U_4$ and $U_5$ are adjacent.

Edge in right graph.

$f(u_1) v_1$ and $f(v_2) = v_3$ are adjacent
$f(u_1) = v_1$ and $f(u_4) = v_3$ are adjacent
$f(u_1) = v_1$ and $f(u_5) = v_4$ are adjacent
$f(u_2) = v_5$ and $f(u_3) = v_2$ are adjacent
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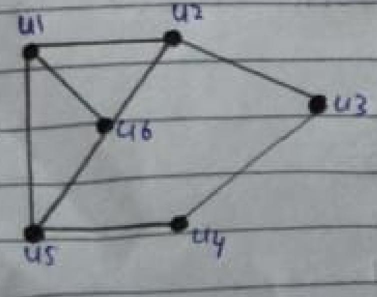
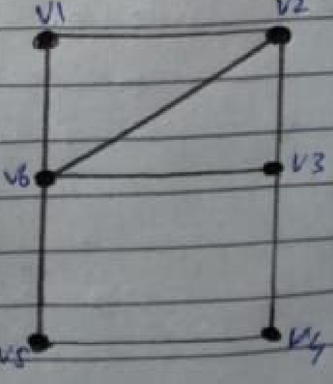


Solution:- let us determine the degree of every vertex in the left graph

degree ($U1$) = 3
degree ($U2$) = 3
degree ($U3$) = 2
degree ($U4$) = 2
degree ($U5$) = 3
degree ($U6$) 3

degree Seauence = 3.3.3.3.2.2.

let us determine next the degree of every vertex in the right graph=

degree ($V1$) = 2
degree ($V2$) = 3
degree ($V3$) = 3
degree ($V4$) = 2
degree ($V5$) = 2
degree ($V6$) = 4

~~Degree Seauence = 4, 3, 3, 2, 2, 2.~~

Isomorphic graph need to have the same number of vertices the same number of edges and the same degree seauences we then note that the two graph do not have the same degree seauence and thus two graph are not isomorphic.

Q3 Are the Simple graphs with the following adjacency matrices Isomorphic

As
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

= let us add all elements in the two matrices. We will represent the number of connect vertex to another vertex which is double the number of edge. A contain 8 ones and thus the graph corresponding to A contains 8 connection B contains 10 ones and thus the simple graph corresponding to A contains 10 connection. Since the number of connection of two graph are not the same the number of edges in the graphs are not the same and then the graph are not isomorphic.

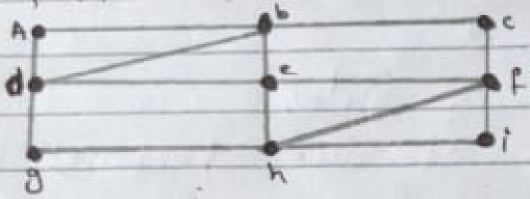
3)
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Sol:- Let us add all the elements in the two matrices, which will represent the number of connection of vertex to another vertexs which is double the number of edge. A contains 8 ones and thus the simple graph corresponding to A contains 8 connection.

B contains 6 ones and thus the simple graph corresponding to A contains connection.

Since the number of connection of two graph are not the same, the number of edge in the graph are not the same and the graph are not Isomorphic.

Q4. Determine wether the given graph has an Euler Circuit. Contruct Such a Circuit when one exist. If no euler circuit exist. determine wether the graph has an Euler Path and Construct Such a Path if one exists.

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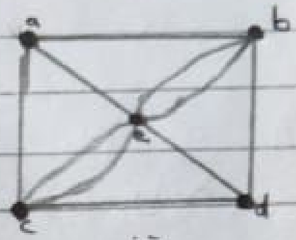



(ii)

sol-> Lets first determine the degree of every Vertex in the given graph-

degree (a) = 2
degree (b) = 4
degree (c) = 2
degree (d) = 4
degree (e) = 4
degree (f) = 4
degree (g) = 2
degree (h) = 4
degree (i) = 2

A graph has an Euler Circuit if and only if each of the othe verleices has an even degree. Since all degree are even. there exists an euler circuits A possible Euler circuits is, a, b, c, f, i, h, g, d, e, b, d, a.

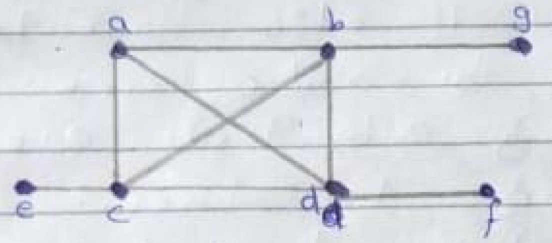
2) Sol: lets first determine the degree of every Vertex in the given graph.

degree(a) = 3
degree(b) = 4
degree(c) = 4
degree(d) = 3
degree(e) = 6

A graph has an Euler circuit if and only if each of the vertaices has an even degree. Since some degree are odd. there is no Euler circuit.
A graph has an Euler Path if and only if there are exactly two vertices who have an odd degree. we note that Vertices a and d have an odd degree and thus an Euler Path exists

A Possible. Euler Path is - a, b, c, ~~b~~ d, c, a, c, c, c, d.

Q5) Determine wether the given graph has a Hamilton circuit. if it does find such a circuit. if it does not. given an argument to show why no such circuit exists.

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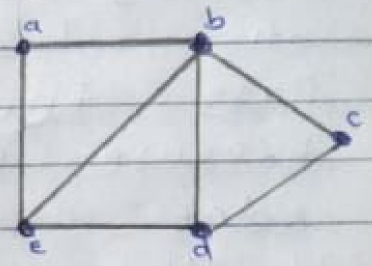


(ii)

Soln– A Hamilton circuit is a simple circuit that pass through every vertex exactly once.

lets first determine the degree of every vertex in the given graph

degree (a) = 3
degree (b) = 4
degree (c) = 4
degree (d) = 4
degree (e) = 1
degree (f) = 1
degree (g) = 1

We then note that Dirac's theorem is not satisfied since some degree are less then $n/2 = 7/2 = 3.5$. but this does not necessarily mean that no Hamilton circuits exists However we do note that there is only one edge ({d, f}) connecting to f and thus any circuit that contains

f needs to Pass through d twice which means that no Circuit can be a Himilton circuit as we Pass through a vertex more than once.
Similary a circuit that pass through all vertices will also have to Pass through b and c twice because c and g only have one edge connecting to them.
Himilton circuit does not exist, because f has only 1 edge connecting to it.

8- lets first determine the degree of every vertex in the given graph

degree (a) = 2
degree (b) = 4
degree (c) = 2
degree (d) = 3
degree (e) = 3

We then note that Diracs theorem is not satisfied since some degree are less then $n/2 = 5/2 = 2.5$ but this does not neccesarily mean that no Hamilton circuit exists. However we do note that the given graph contains the cycle $C_5$ and the cycle $C_5$ within the give graph forms Hamilton circuit as the circuit will pass through all vertices exactly once.
A Possible Hamilton circuit is thus the Path of $C_5$: a,b,c,d,e,a.