Subject :- differential equation

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Q 1

(i) Order of matrix A is $m \times p$
order of matrix B is $p \times n$
$\Rightarrow$ order of matrix AB = $m \times n$

(ii) The number of Non-zero rows is on echelon form is called Rank of the matrix.

(iii) if $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

$\Rightarrow |B| = 0$ i.e singular

$a = ?$

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$|| = 1 \times a - 2 \times 4$$

$$|B| = a - 8$$

But $|B| = 0$

$\Rightarrow$ $a - 8 = 0$

$\Rightarrow$ $a = 8$

(iv) $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$, $|A| = ?$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$|| = 2i \times (-i) - i \times i$

$|| = -2i^2 - i^2$

$|| = -2(-1) - (-1)$

$|| = +2 + 1$

$|A| = 3$

(v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

it is called scaler matrix.

(vi) $\frac{dy}{dx} + 2xy = y$?

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1 - 2x)$

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$$\int \frac{1}{y}\, dy = \int (1-2x)\, dx$$

$$\ln|y| = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln|y| = x - x^2 + c \quad \text{Ans}$$

(vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^3} \text{ is}$$

Order = $\underline{1}$

Degree = $\underline{3}$

(viii) The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

Order = Two

Degree = One

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(ix) The differential equation

$$2\frac{dy}{dx} + x^2 y = 2x+3 , y = (0)$$
$= 5$ is ____

Solution:-

$$2y' + x^2 y = x^2+3 \quad , y(0) = 5$$

$$y' + \left(\frac{x^2}{2}\right) y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right) y = \frac{1}{2}\left(x^2+3\right)$$

(x)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} \begin{matrix} \text{By} \\ R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{vmatrix} 1 & a & a^2 \\ 0 & (b-a) & (b-a)\ b+a) \\ 0 & (c-a) & (c-a)(c+a) \end{vmatrix}$$

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$$(b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & (b+a) \\ 0 & 1 & (c+a) \end{vmatrix}$$

$$(b-a)(c-a)\left[1\begin{vmatrix} 1 & (b+a) \\ 1 & (c+a) \end{vmatrix} - 0 + 0\right]$$

$$(b-a)(c-a)\left[1(c+a) - 1(b+a)\right]$$

$$(b-a)(c-a)(c+\cancel{a} - b - \cancel{a})$$

$$= (b-a)(c-a)(c-b) \text{ Ans.}$$

Q No 2

Part (i)

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$= abc\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

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$$= abc \begin{vmatrix} 1 & 0 & 0 \\ a & (b-a) & c-a \\ a^2 & (b^2-a^2) & c^2-a^2 \end{vmatrix}$$

$$= (abc)(b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & (b+a) & (c+a) \end{vmatrix}$$

$$= (abc)(b-a)(c-a) \left[1 \begin{vmatrix} 1 & 1 \\ (b+a) & (c+a) \end{vmatrix} - 0 + 0 \right]$$

$$= (abc)(b-a)(c-a) \left[1(c+a) - 1(b+a) \right]$$

$$= (abc)(b-a)(c-a)(c + \cancel{a} - b - \cancel{a})$$

$$= (abc)(b-a)(c-b)$$ Ans.

Q No 2

Part (B)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

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Sol:
$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$

Characteristic equ $\rightarrow |A - \lambda I| = 0$ (A)

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now take determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand By R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow (B)$$

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Again:-

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda) \left[\left((3-\lambda)(2-\lambda) - (-1)(-1) \right) + 1 \left((-1)(2-\lambda) - (-1)(-1) \right) - 1 \left((-1)(-1) - (-1)(3-\lambda) \right) \right]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$\Rightarrow 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$\Rightarrow \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \rightarrow a$$

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$$\Rightarrow \quad -1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow \quad -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow \quad -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow \quad -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \quad \boxed{-\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow \quad -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow \quad -\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow \quad -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$\Rightarrow \quad -(3-\lambda+\lambda^2-5\lambda+5)$$

$$\Rightarrow \quad -\lambda^2+5\lambda-5-3+\lambda$$

$$= \quad \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

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put a, b and c in B

$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8\lambda^2$
$+6\lambda-8$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2$
$-8\lambda-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2$
$-36\lambda-8\lambda+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

BY synthetic division
we get

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$(\lambda=0)$

$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$

$\lambda^2-8\lambda+16=0$

BY factorization method

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)$

$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=4}$ Ans

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Q No 3

Solution: Data,

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$\frac{(x^2+3y^2)}{2xy} = \frac{dy}{dx}$$

$$\left(\frac{x^2}{2xy} + \frac{3y^2}{2xy}\right) = \frac{dy}{dx}$$

$$\left(\frac{x}{2y} + \frac{3}{2}\cdot\frac{y}{x}\right) = \frac{dy}{dx}$$

Let $u = \frac{y}{x}$ OR $\frac{1}{u} = \frac{x}{y}$

$\Rightarrow y = ux$

Diff. w.r.to "x" on b.s

$$\frac{dy}{dx} = u\cdot 1 + x\frac{du}{dx}$$

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$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\frac{1}{2} \cdot \frac{1}{u} + \frac{3}{2}u = u + x\frac{du}{dx}$$

$$\frac{1}{2u} + \frac{3}{2}u - \frac{u}{1} = x\frac{du}{dx}$$

$$\frac{1+3u^2-2u^2}{2u} = x\frac{du}{dx}$$

$$\frac{1+u^2}{2u} = x\frac{du}{dx}$$

$$\int \frac{1}{x}\,dx = \int \frac{2u}{1+u^2}\,du$$

$$\ln|x| = \ln|1+u^2| + \ln|c|$$

$$\ln|x| = \ln|1+u^2|\,|c|$$

$$x = |1+u^2)\,|c|$$

where $x=2, y=6,$

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$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\frac{1}{2}\cdot\frac{1}{u} + \frac{3}{2}u = u + x\frac{du}{dx}$$

$$\frac{1}{2u} + \frac{3u}{2} - \frac{u}{1} = x\frac{du}{dx}$$

$$1 + 3u^2 - 2u^2 = x\frac{du}{dx}$$

$$\frac{1+u^2}{2u} = x\frac{du}{dx}$$

$$\int \frac{1}{x}\,dx = \int \frac{2u}{1+u^2}\,du$$

$$\ln|x| = \ln|1+u^2| + \ln|c|$$

$$\ln|x| = \ln|1+u^2|\,|c|$$

$$x = |1+u^2|\,|c|$$

where $x = 2, y = 6$, But $u = \frac{y}{x}$

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PS#14

$x = \left|1 + \frac{y^2}{x^2}\right| (c)$

$2 = \left|1 + \frac{6^2}{2^2}\right| c$

$2 = \left|1 + \frac{\cancel{36}^{9}}{\cancel{4}}\right| c$

$2 = |1 + 9| c$

$2 = 10 c$

$c = \frac{\cancel{2}^{1}}{\cancel{10}_{5}}$

$c = \frac{1}{5}$

$x = \left|1 + \frac{y^2}{x^2}\right| \frac{1}{5}$

$5x = \left|\frac{x^2 + y^2}{x^2}\right|$ Ans.

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