ID : 14192

Name : Salman Khan

Differential Equations

Assignment :

Q1

(a): $x' = \sqrt{x}$

Sol:

step 1:

Given: $x' = \sqrt{x}$

Here x is the dependent Variable let is take t as the independent variable.

Then differential equation becomes

$$\frac{dx}{dt} = \sqrt{x}$$

~~Second~~ step 2: Consider

$$\frac{dx}{dt} = \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = dt$$

on integration on both Side, we get

$$\int \frac{dx}{\sqrt{x}} = dt$$

$$\int x^{-\frac{1}{2}} = \int dt$$

$$-\frac{1}{2} x^{-\frac{1}{2}-1} = t + c$$

$$-\frac{1}{2x^{3/2}} = t + c$$

This is the required Solution

(2)

(b) $x' = e^{-2x}$

Sol:

step 1:

Consider the given differential equation

$x' = e^{-2x}$

it can be written as,

$\frac{dx}{dt} = e^{-2}$

Multiply both Sides by dt

$dx = e^{-2x}\, dt$

Divide both side by dt.

$\frac{dx}{e^{-2x}} = dt$

it can be written as

$e^{2x}\, dx = dt$

step 2:

The equation $e^{2x}\, dx = dt$ in Variable Separble form

Integrated both Side

$\Rightarrow \int e^{2x}\, dx = \int dt$

$\Rightarrow \frac{e^{2x}}{2} = t + c$

Multiply both Side by 2

$e^{2x} = 2(t+c)$

Taking natural logrith function from both Side,

$= \ln(e^{2x}) = \ln(2t + 2c)$

$= 2x = \ln(2t + C_1)$

$= x = \frac{\ln(2t + C_1)}{2}$

The Solution of the given differential equat^n is $x = \frac{\ln(2t + C_1)}{2}$

(C) $y' = 1 + y^2$

Sol:.

Step 1:. Here we use the method of Separation of Variable to find the general Solution of the differential equation

$$y' = 1 + y^2$$

Step 2:. Here

$$y' = 1 + y^2$$

$$= \frac{dy}{dx} = 1 + y^2 \quad \left[\because y' \frac{dy}{dx}\right]$$

$$= dy = (1 + y^2)\, dx$$

$$= \frac{dy}{1+y^2} = dx$$

integrating both Side, we get

$$\int \frac{dy}{1+y^2} = \int dx + c \quad \left[\because \int \frac{dx}{1+y^2} = \tan^{-1}(y)\right]$$

$$= \tan^{-1}(y) = x + c$$

$$\Rightarrow \boxed{y = \tan(x+c)}$$

Step 3: Hence the general Solution of the differential equation $y' = 1 + y^2$ is

$$\boxed{y = \tan(x+c)}$$

(d): $u' = \frac{1}{5-2u}$

Sol.

Step 1: Given $u' = \frac{1}{5-2u}$

Compute general equation as follow: Rewrite the given differential equation as.

$$\frac{du}{dv} = \frac{1}{5-2u}$$

$$(5-2u)\,du = dv$$

Step 2:

Now integrate on both side

$$\int(5-2u)\,du = \int dv$$

$$5u - \frac{2u^2}{2} + C = v + C$$

$$5u - u^2 = v + C$$

on Further Simplification

$$u^2 - 5u = C - v$$

$$\left[u^2 - 5u + \frac{25}{4} - \frac{25}{4}\right] = C - v$$

$$\left(u - \frac{5}{2}\right)^2 = C - v$$

$$\left(u - \frac{5}{2}\right) = \sqrt{C - v}$$

$$\boxed{u = \frac{5}{2} + \sqrt{C-v}}$$

Thus, the general Solution is $u = \frac{5}{2} + \sqrt{C-v}$, where C is costant

(e) $x' = au + b$, $a, b > 0$

Sol:

Step 1:

given

$$x' = au + b \quad \cdots(1) \quad a > 0,\ b > 0$$

Step 2:

Equation (1) Can be written as

$$\frac{dx}{du} = au + b$$

by Variable of equation

$$dx = (au + b)\,du.$$

Integration on both side

$$\int dx = \int (au+b)\, du + c$$

$$\int dx = \int au\, du + \int b\, du + c$$

$$\int dx = a\int u\, du + b\int du + c$$

$$\boxed{x = a\frac{u^2}{2} + bu + c}$$

(f): $Q' = \frac{Q}{4+Q^2}$

Sol:

Step 1:

Given differential equation

$$Q' = \frac{Q}{4+Q^2}$$

let the independent variable be t and thus we have

$$Q' = \frac{dQ}{dt}$$

So given differential equation becomes

$$\frac{dQ}{dt} = \frac{Q}{4+Q^2}$$

Rewrite the above differential equation as

$$\frac{(4+Q^2)}{Q}\, dQ = dt$$

Hence, differential Equation $\frac{(4+Q^2)}{Q}\, dQ = dt$ is Variable Separable equation

Step 2:

Proceed using the method of Separation of variable & obtain the solution of the differential equation $\frac{(4+Q^2)}{Q}\, dQ = dt$ as follow

$$\int \frac{(4+\varphi^2)}{\varphi} d\varphi = \int dt$$

$$\int \left(\frac{4}{\varphi} + \frac{\varphi^2}{\varphi}\right) d\varphi = \int dt$$

$$\int \left(\frac{4}{\varphi} + \varphi\right) d\varphi = \int dt$$

$$\int \frac{4}{\varphi} d\varphi + \int \varphi d\varphi = \int dt$$

$$\boxed{4 \ln|\varphi| + \frac{\varphi^2}{2} = t + C}$$ Ans

(G): $x' = e^{x^2}$

Sol:

step 1:

Given Equation $x' = e^{x^2}$

Rewrit equation $\frac{1}{e^{x^2}} \frac{dx}{dt} = 1$

$$\frac{1}{e^{x^2}} dx = dt$$

Integrating both side

$$\int \frac{1}{e^{x^2}} dx = \int 1 dt$$

$$\int e^{-x^2} dx = \int 1 dt$$

Step 2: $\int \frac{1}{e^{x^2}} dx = \int 1 dt$

$\int e^{-x^2} dx = \int 1 dt$

Multiply & divide by $\sqrt{\pi}$

$\int \frac{1}{e^{x^2}} dx = \int 1 dt$

$\sqrt{\pi} \int \frac{1}{\sqrt{\pi}} e^{-x^2} dx = \int 1 dt$

Multiply & divided by 2.

$\sqrt{\frac{\pi}{2}} \int 2 \frac{1}{\sqrt{\pi}} e^{-x^2} dx = \int 1 dt \quad \cdots (1)$

the integral $\int \frac{2}{\sqrt{\pi}} e^{-x^2} dx$ is the erf(x)

therefore, (1) reduces to $\frac{\sqrt{\pi}}{2}$ erf(x) = t+c

where C is the constant of integration

The answe. is : $\boxed{\sqrt{\frac{\pi}{2}} \text{ erf}(x) = t + c}$

(H): $y' = r(a-y)$

Sol:

Step 1:

Given $y' = r(a-y)$

Step 2:

Given differential equation $y' = r(a-y)$

$y' = r(a-y)$

$\frac{dy}{dt} = r(a-y)$

$\frac{dy}{(a-y)} = r\,dt$

integrating we get

(8)

$$\int \frac{dy}{(a-y)} = \int r\,dt$$

$$-\ln(a-y) = rt + c$$

$$\ln(a-y) = -rt - c$$

Step 3:

Solve the equation $\ln(a-y) = -rt-c$ for y as shown below.

$$\ln(a-y) = -rt - c$$

$$e^{\ln(a-y)} = e^{-rt-c}$$

$$a-y = e^{-rt-c}$$

$$\boxed{y = -e^{-rt-c} + a}$$

Therfore

The general equation or the differential equation $y' = r(a-y)$ is $\boxed{y = -e^{-rt-c} + a}$