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CLASS : SECTION - B (2nd Semester)

PAPER : CALCULUS

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QUESTION: (1)

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t + 3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

(a) state any point of discontinuity

(b) Find, if they exist

(i) $\lim_{t \to 3} g$

SOLUTION:

(a) To check possibility of the discontinity of the function is at $t = 0$ & 4

First at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L

$$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply limits

$= 1 + 0^2 + 2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t + 3$

$= \lim_{h \to 0} 2(1-h) + 3$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

For R.H.L

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$

$= \lim_{h \to 0} 2 + 2h + 3$

Apply limits

$= 2 + 2(0) + 3 \Rightarrow 5$

FOR L.H.L

$\lim_{h \to 0} g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

point of discontinuity is at $t=4$

(b) $\lim_{t \to 3} g$

FOR $g(t) = t^2$

R.H.L

$\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limit

$= 1 + (3)^2 + 2(3)$

$= 1 + 9 + 6$

$= 10 + 6$

$= 16$

L.H.L

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L

Don't exist since L.H.L is negative (-ve).

QUESTION: 2

(i) Find the Maclaurin's series for $y(x) = x^2 + \sin x$

SOLUTION: 2

$$F(x) = F(0) + x F'(0) + \frac{x^2}{2!} F''(0) + \frac{x^3}{3!} F'''(0)$$

Let $y(x) = x^2 + \sin x$

$y'(x) = 2x + \cos x$

$y''(x) = 2 - \sin x$

$y'''(x) = -\cos x$

put $x = 0$ in all Function

$y(0) = 0$

$y'(0) = 1$

$y''(0) = 2$

$y'''(0) = -1$

Putting value in formula

$$= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1)$$

$$= 0 + x + x^2 - \frac{x^3}{6}$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6}$$

QUESTION:(3)

(i) Find y'' given

$$1 + xy = x^3 + y^2$$

SOLUTION:

$$1 + xy = x^2 + y^2 \quad \text{———} \quad (1)$$

diff equation (1) b/s w.r.t "x"

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$= 0 + \left(x \cdot \frac{d}{dx}y + y \cdot \frac{d}{dx}x\right) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot \frac{d}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$= x \cdot y' + y = 2x + 2y \cdot y$$

$= x \cdot y' - 2y \cdot y' = 2x - y$

$= y'(x - 2y) = (2x - y)$

$$= y' = \frac{2x - y}{x - 2y}$$

$$y' = \frac{2x - y}{x - 2y}$$

Differ again w.r.t "x"

$$y'' = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$$

$$= \frac{(x-2y)\, d/dx\,(2x-y) - (2x-y)\, d/dx\,(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2)\left(-\frac{dy}{dx}\right) - (2x-y)\left(1 - 2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(2x-4y)(-y') - (2x - 4x\frac{dy}{dx} - y + 2yy')}{(x-2y)^2}$$

$= y''(x-2y)^2 = 2xy' + 2yy' - 2x + y$

$y''(x-2y)^2 = \left(\frac{2x-y}{x-2y}\right)(2x-2y) - 2x + y$

$$y'' = \frac{\left(\frac{2x-y}{x-2y}\right)(2x+2y) - 2x+y}{(x-2y)^2}$$

$$y'' = \frac{(2x-y)/(x-2y).(2x+2y) - 2x+y}{(x-2y)^2}$$

(2) Find y' by using Logarithmic differentiation $y = x^3(1+x)^9 e^{6x}$

SOLUTION:

Taking ln on b/s

$$\ln y = \ln\left(x^3(1+x)^9 . e^{6x}\right)$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x\ln e$$

Diff w.r.t x

$$\frac{d}{dx}\ln y = \frac{d}{dx} 3\ln x + \frac{d}{dx} 9\ln(1+x) + \frac{d}{dx} 6x \ln e$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}}.6$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 . e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$$