Department of Electrical Engineering
Final term exam
Date: 25/09/2020

Course Details

Course Title: Complex & Multivariable Calculus **Module:** 03
Instructor: **Mujtaba ihsan** **Total Marks:** 50

Student Details

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Q1.	(a)	$A = x^2 y^4 z^3\, i - 3y^2 z\, j + 4xz^2 k$ express $\nabla \times (\nabla \times A)$	Marks 08 +10
	(b)	**Extend** $\iint_1^4 (2x + 6x^2 y)\, dydx$	CLO 2
Q2.		**Express** the equation of the plane passing through the point (5, -2, 4) that is perpendicular to the plane 3x + y - 6z + 8 = 0	Marks 06 CLO 2
Q3.		Given a = < 2, -1, 6 > and b = < -3, 5, 1 > **express** a x b	Marks 08 CLO 2
Q4.		**Estimate** the angle between the plane 4x + 2y - 6z = 10 and xz plane.	Marks 08 CLO 2
Q5.		Give all values of $\sin^{-1}\sqrt{5}$	Marks 10 CLO 1

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Subject ComPlex

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Q1 (b) Extend $\int_1 \int_1^4 (2x+6x^2y)\,dy\,dx$

Sol:

$$\int_1 \int_1^4 (2x+6x^2y)\,dy\,dx$$

$$\int \left[\int_1^4 (2x+6x^2y)\,dy\right] dx$$

$$\int \left[2x+6\frac{x^2y^2}{2}\Big|_1^4\right] dx$$

~~Now~~ we write ~~dx~~

$$\cancel{\int\left[2x+6x^2\frac{(1)^2}{2}\right]}$$

$$\int \left[2x+\frac{6x^2(4)^2}{2}-2x+\frac{6x^2(1)^2}{2}\right] dx$$

$$\int \left[2x+\frac{6x^2(16)}{2}-2x+\frac{6x^2}{2}\right] dx$$

$$\int \left[2x+48x^2-2x+3x^2\right] dx$$

Now w.r.t dx

$$\int \left[2x + 48x^2 - 2x + 3x^2 \right] dx$$

$$\left[\frac{2x^2}{2} + \frac{48x^3}{3} - \frac{2x^2}{2} + \frac{3x^3}{3} \right]$$

$$= \cancel{x^2} + 16x^3 - \cancel{x^2} + x^3$$

$$\boxed{= 17x^3}$$

Q3:- Given $a = \langle 2, -1, 6 \rangle$ $b = \langle -3, 5, 1 \rangle$

Express $a \times b$.

Sol:- $a \times b = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$$= i \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - j \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + k \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$= i \begin{vmatrix} -1 & 6 \\ 5 & 1 \end{vmatrix} - j \begin{vmatrix} 2 & 6 \\ -3 & 1 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ -3 & 5 \end{vmatrix}$$

$$= i\left((-1 \times 1) - (5 \times 6)\right) - j\left((2 \times 1) - (-3 \times 6)\right) + k\left((2 \times 5) - (-3 \times -1)\right)$$

$$= i(-1-30) - j(2+18) + k(10-3)$$

$$= i(-31) - j(20) + k(7)$$

$$\boxed{= -31i - 20j + 7k}$$

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Q4:: Angle b/w $4x+2y-6z=10$

Sol::

$$n_1 \cdot n_2 = |n_1||n_2| \cos\theta$$

$$10 = \sqrt{4}\sqrt{-6} \cos\theta$$

$$\cos\theta = \frac{(4)(-6)}{|4||-6|}$$

$$\theta = \cos^{-1} \frac{-24}{24}$$

$$\boxed{\theta = \cos^{-1} -1}$$

$$\boxed{\theta = 180^\circ}$$

Q5:- Give all values of $\sin^{-1}\sqrt{5}$

Sol:- From equation

$$\sin^{-1} z = -i \ln\left[iz + (1-z^2)^{1/2}\right]$$

$$\sin^{-1}\sqrt{5} = -i \ln\left[\sqrt{5}i + \left(1-(\sqrt{5})^2\right)^{1/2}\right]$$

$$\left(1-(\sqrt{5})^2\right)^{1/2} = (-4)^{1/2} = \pm 2i$$

$$\sin^{-1}\sqrt{5} = -i \ln\left[(\sqrt{5} \pm 2)i\right]$$

$$= -i\left[\log_e(\sqrt{5} \pm 2) + \left(\frac{\pi}{2} + 2n\pi\right)i\right]$$

$$n = 0, \pm 1, \pm 2, \ldots$$

So

$$\log_e(\sqrt{5}-2) = \log_e \frac{1}{\sqrt{5}+2}$$

$$= -\log_e(\sqrt{5}+2)$$

Thus for $n = 0, \pm 1, \pm 2 \ldots$

$$\sin^{-1}\sqrt{5} = \frac{\pi}{2} + 2n\pi \pm i \log_e(\sqrt{5}+2)$$

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