



**Department of Electrical Engineering**  
**Electro Magnetic Field Theory**

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**QUESTION:(1):**

**Solve the following short Question.**

**Q (A):**

**Solution:**

$$\rho = \sqrt{(x^2+y^2)}$$

$$= \sqrt{40}$$

$$= 6.325$$

$$\Phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}(-6/2)$$

$$= -71.57$$

$$z = 3$$

$$(6.325, -71.57, 3)$$

**Q (B):**

**Solution:**

$$r = \sqrt{(x^2+y^2+z^2)}$$

$$= \sqrt{50}$$

2

$$= 7.07$$

$$\Theta = \cos^{-1}(z/r)$$

$$= \cos^{-1}(5/5\sqrt{2})$$

$$= 45^\circ$$

$$\Phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}(4/3)$$

$$= 53^\circ.$$

$$(7.07, 45^\circ, 53^\circ)$$

**Q (C):**

**Solution:**

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{14}$$

$$= 3.74$$

$$\Theta = \cos^{-1}(z/r)$$

$$= \cos^{-1}(-1/3.74)$$

$$= 105.5^\circ$$

$$\Phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}(3/2)$$

$$= 56.31^\circ.$$

$$(3.74, 105.5^\circ, 56.31^\circ)$$

**Q (D):**

**Solution:**

$$x = r \sin \theta \cos \varphi = 4 \sin 25^\circ \cos 120^\circ = -0.845$$

$$y = r \sin \theta \sin \varphi = 4 \sin 25^\circ \sin 120^\circ = 1.462$$

$$z = r \cos \theta = 4 \cos 25^\circ = 3.625$$

$$(-0.845, 1.462, 3.625)$$

**Q (E):**

**Solution:**

Before the charges are brought into contact,  $F = 11.234 \mu\text{N}$ .

After charges are brought into contact and then separated, charge on each sphere is,  $(q_1 + q_2)/2$

$$= 0.5\text{nC}$$

On calculating the force with  $q_1 = q_2 = 0.5\text{nC}$ ,

$$F = 1.404\mu\text{N}.$$

**Q (F):**

**Solution:**

$$\begin{aligned} F &= q_1q_2/(4\pi\epsilon_0r^2) \\ &= -2 \times 9/(10^{-9} \times 12) \\ &= -18 \times 10^9 \end{aligned}$$

**Q (G):**

**Solution:**

$$\begin{aligned} E &= Q/(4\pi\epsilon_0r^2) \\ Q &= (4000 \times 0.32)/(9 \times 10^9) \\ &= 4 \times 10^{-8} \\ C &= 4 \end{aligned}$$

**Q (H):**

**Solution:**

$$\begin{aligned} F &= q_1q_2/(4\pi\epsilon_0r^2) \\ &\text{substituting } q_1, q_2 \text{ and } F, \\ r^2 &= q_1q_2/(4\pi\epsilon_0F) \end{aligned}$$

We get  $r = 0.09\text{m}$ .

**QUESTION 2: (A)**

**Answer:**

$$A = \sqrt{3}ix + iy$$

$$|A| = 2$$

$$B = 2ix$$

$$|B| = 2$$

$$A \cdot B = 2\sqrt{3}$$

Now,

$$A \cdot B = |A| |B| \cos \theta_{AB}$$

$$\cos \theta_{AB} = \frac{A \cdot B}{|A| |B|}$$

$$\theta_{AB} = \cos^{-1} \left( \frac{2\sqrt{3}}{(2 \times 2)} \right)$$

$$\theta_{AB} = 30^\circ$$

### QUESTION 2: (B)

Answer: (a)

$$F = ax^3 + by^3z$$

$$\Delta f = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) (ax^3 + by^3z)$$

$$\Delta f = \left( \frac{\partial}{\partial x} ax^3 + \frac{\partial}{\partial y} by^3z + \frac{\partial}{\partial z} by^3z \right)$$

$$\Delta f = 2axi + 3by^2j + by^3k$$

### QUESTION 2: (B)

Answer: (b)

In case of cylindrical;

$$\Delta = \left( \frac{\partial}{\partial r} \right) \hat{r} + \left( \frac{1}{r} \right) \left( \frac{\partial}{\partial \theta} \right) \hat{\theta} + \left( \frac{\partial}{\partial z} \right) \hat{z}$$

$$\Delta f = \left\{ \left( \frac{\partial}{\partial r} \right) \hat{r} + \left( \frac{1}{r} \right) \left( \frac{\partial}{\partial \theta} \right) \hat{\theta} + \left( \frac{\partial}{\partial z} \right) \hat{z} \right\} \{ ar^2 \sin \theta + brz \cos 2\theta \}$$

$$\Delta f = \frac{\partial}{\partial r} (ar^2 \sin \theta + brz \cos 2\theta) \hat{r} + \left( \frac{1}{r} \right) \left( \frac{\partial}{\partial \theta} \right) (ar^2 \sin \theta + brz \cos 2\theta) \hat{\theta} +$$

$$\frac{\partial}{\partial z} (brz \cos 2\theta) \hat{z}$$

$$(2ar \sin \theta + brz \cos 2\theta) \hat{r} + \left( \frac{1}{r} \right) (ar^2 \cos \theta - 2brz \sin 2\theta) \hat{\theta} + br \cos 2\theta \hat{z}$$

### QUESTION 3:

Answer:

$$r = bax + 0.ay + 0.az$$

$$r_1 = 0.ax + 0.ay + 0.az$$

$$r_2 = 0.ax + 0.ay + 0.az$$

$$r_3 = 0.ax + (-a) ay + 0.az$$

$$r - r_1 = bax + 0.ay + 0.az$$

$$r - r_2 = bax - a.ay + 0.az$$

$$r - r_3 = b.ax + a.ay + 0.az$$

$$\begin{cases} r - r_1 & = b \\ r - r_2 & = \sqrt{(b^2 + a^2)} \\ r - r_3 & = \sqrt{(b^2 + a^2)} \end{cases}$$

Now,

$$E = Q / 4\pi\epsilon_0 r$$

For change 1,

$$\begin{aligned} E_1 &= -Q / 4\pi\epsilon_0 (r - r_1) / (|r - r_1|) \\ &= -Q / 4\pi\epsilon_0 \{ bax / b \} \{ 1/b^2 \} \end{aligned}$$

For change 2,

$$E_2 = 2Q / 4\pi\epsilon_0 \{ bax - aay / (\sqrt{b^2 + a^2}) \} \{ 1 / (\sqrt{b^2 + a^2}) \}$$

For change 3,

$$E_3 = 2Q / 4\pi\epsilon_0 \{ bax - aay / (\sqrt{b^2 + a^2}) \} \{ 1 / (\sqrt{b^2 + a^2}) \}$$

$$E = E_1 + E_2 + E_3$$

$$E = [-Q / 4\pi\epsilon_0 \{ bax / b \} \{ 1/b^2 \}] + [2Q / 4\pi\epsilon_0 \{ bax - aay / (\sqrt{b^2 + a^2}) \} \{ 1 / (\sqrt{b^2 + a^2}) \}] + [2Q / 4\pi\epsilon_0 \{ bax - aay / (\sqrt{b^2 + a^2}) \} \{ 1 / (\sqrt{b^2 + a^2}) \}]$$

$$E = Q / 4\pi\epsilon_0 [ 1 / \{ bax / b \} \{ 1/b^2 \}] + [ 2 / \{ bax - aay / (\sqrt{b^2 + a^2}) \} \{ 1 / (\sqrt{b^2 + a^2}) \}] +$$

$$[ 2 / \{ bax - aay / (\sqrt{b^2 + a^2}) \} \{ 1 / (\sqrt{b^2 + a^2}) \}]$$

**Thank You.**