Summer-20 Final Term Assignment

Subject: Discrete Structure

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Question No. 1: **(10)**

a) Find the 36th term of the arithmetic sequence whose 3rd term is 7 and 8^{th} term is 17.

Answer

let a be the first term and d be the common difference of the arithmetic sequence. Then

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$ and

$a_8 = a + (8-1)d$

Given that $a_3 = 7$ and $a_8 = 17$ There for

$7 = a + 2d$ .... (1) and

$17 = a + 7d$ .... (2)

Subtracting (1) from (2) we get

$10 = 5d$

$d = 2$

Subtracting $d = 2$ in (1) we have

$7 = a + 2(2)$

which gives $a = 3$

Thus $a_n = a + (n-1)d$

$a_n = 3 + (n-1)2$ (using value of a and

hence the value of 36^{th} term is

$a_{36} = 3 + (36-1)2$

$= 3 + 70$

$= 73$

Question No. 2: **(10)**

Find ***fog*(x)** and ***gof(x)*** of the functions f(x) = 2x + 3 and $g(x) = -x^2 + 5$

Answer

Given $f(x) = 2x+3$ and $g(x) = -x^2+5$,

$(fog)x = f(g(x)) = f(-x^2+5)$

$= 2(\quad)+3$ insert the input

formula $= 2(-x^2+5)+3$

$= -2x^2+10+3$

$= -2x^2+13$ Ans

Question No. 3: **(10)**

Prove by mathematical induction that the statement is true for all integers n ≥ 1 **(10)**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Answer

Solution

Step

P(1) is true

for n = 1

L.H.S of $P(1) = (1)^2 = 1$

R.H.S of $P(1) = \frac{1(1+1)(2(1)+1)}{6}$

$= \frac{(1)(2)(3)}{6} = \frac{6}{6} = 1$

So L.H.S = R.H.S of P(1) Hence P(1) is true

Suppose P(K) is true for some integer K≥

$1^2+2^2+3^2+\cdots K^2 = \frac{K(K+1)(2K+1)}{6}$

To prove P(K+1) is true ie;

$1^2+2^2+3^2+\cdots+(K+1)^2 = \frac{(K+1)(K+1+1)(2(K+1)}{6}$

Consider LHS of above equation

$1^2+2^2+3^2+\cdots+(K+1)^2$

$= 1^2+2^2+3^2+\cdots+K^2+(K+1)^2$

$= \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= (K+1)\left[\frac{K(2K+1)}{6} + (K+1)\right]$

$$= (k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= \cancel{(k+1)}\ \cancel{[}$$

$$\frac{(k+1)(2k^2+7k+6)}{6}$$

$$\frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1}{6}$$

Question No. 4: **(10)**

Discuss different types of relations with example in detail.

Answer

Binary Relation Ship

Let A and B be sets. The binary relation R from A to B is a subset of $A \times B$

Example

Let $A = \{1,2\}$, $B = \{1,2,3\}$

Domain of a Relation

The domain of a relation R from A to B is the set of all first elemet of the orderd pairs.
R denoted by Dom (R)
Symbolically $\text{Dom } R = \{a \in A \mid (a,b) \in R\}$

Range of a Relation

The range of a relation R from A to B is the set of all second elemet of the orderd pair which belong Ran(A)
Symbolically $\text{Ran}(R) = \{b \in B \mid (a,b) \in R\}$

Reflexive Relation

Let R be a relation on a set A. R is reflexive if and only if, for all $a \in A$, $(a,a) \in R$ That is each elemet of A is related to itself.

Symmetric relation

Let R be a relation on a set A. R is symmetric if and only if for all a, b ∈ A if (a, b) ∈ R then (b, a ∈ R.

Transitive relation

Let R be a relation on a set A. R is transitive if and only if for all a, b, c ∈ A if (ab) ∈ R and (b, c) ∈ R then (a, c) ∈ R.

Question No. 5 **(10)**

Suppose that an automobile license plate has three letters followed by three digits.

a. How many different license plates are possible?

b. How many license plates could begin with A and end on 0.

c. How many license plates begin with PQR

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Question 5

26	26	26	10	10	10

$= 26 \times 26 \times 26 \times 10 \times 10 \times 10$

$= 17576000$

A					0

$= 1 \times 26 \times 26 \times 10 \times 10 \times 1$

$= 67600$

P	Q	R			

$= 10 \times 10 \times 10$

$= 1000$

Thanks