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QNo 1

Sol #

$$P = (\overset{x_1}{4}, \overset{y_1}{1}, \overset{z_2}{3}), \quad Q = (\overset{x_2}{1}, \overset{y_2}{2}, \overset{z_2}{4})$$

Now first we find the distance between P and G

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|PQ| = \sqrt{(1-4)^2 + (2-1)^2 + (4-3)^2}$$

$$= (-3)^2 + (1)^2 + (1)^2$$

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$$= \sqrt{9+1+1} = \sqrt{11}$$

$$\boxed{|PQ| = \sqrt{11}}$$

Now find Position vector of the point dividing PQ in the ratio of 1:3

$$a : p = 1 : 3$$

$$\vec{r} = \frac{b\vec{P} + a\vec{a}}{b+a}$$

$$= \frac{3(4i+j+3k)+1(\hat{2}+2\hat{j}+4\hat{k})}{3+1}$$

$$= \frac{12\hat{i}+3\hat{j}+9K+\hat{i}+2\hat{j}+4\hat{K}}{4}$$

$$\vec{r} = \frac{13}{4}\hat{i} + 5/4\hat{j} + 13/4\hat{K}$$ Ans

(Q No 2)

Evaluate $\int \frac{4x^3+10x+4}{2x^2+x}$

Sol # $\int \frac{4x^3+10x+4}{2x^2+x}$

First we decompose into the partial fraction..

$\left[\frac{2x^2+4}{x(2x+1)}\right]$

$$\frac{4x^3+10x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \quad \text{--- (1)}$$

$$4x^3+10x+4 = A(2x+1)+Bx \quad \text{--- (2)}$$

Put $x=0$ in (2)

$4 = A$ $\quad \boxed{A=4}$

or

Put $x=-\frac{1}{2}$ in (2) we have

$$4\left(-\frac{1}{2}\right)^3+10\left(-\frac{1}{2}\right)+4 = A(0)+B\left(-\frac{1}{2}\right)$$

$$-\frac{1}{2}-5+4 = -\frac{1}{2}B$$

$-B = -1-2 = \boxed{B=3}$

① $\Rightarrow$

$$4x^3+10x+4 = \frac{4}{x} + \frac{3}{2x+1}$$

Facting integration on Both Side.

$$\int \frac{4x^3+10x+4}{x(2x+1)}dx = \int \frac{4}{x}dx + \int \frac{3}{2x+1}dx$$

$$= 4\ln|x| + \frac{1}{2}\,{}^{3}\ln(2x+1)+c$$

$$\int \frac{4x^2+10x+4}{=(2x+1)} = 4\ln|x| + \frac{3}{2}\ln|2x+1|+c$$

Ans.

(Q No 3 a)

$$\int_0^2 x^2 e^x \, dx$$

$u = x^2 \quad du = 2x$
$v = e^x \quad dv = e^x$

Sol $\displaystyle\int_0^2 x^2 \cdot e^x \cdot dx$

$$= \left[x^2 \cdot e^x \Big|_0^2 - \int_0^2 e^x \cdot 2x \right]$$

$$= x^2 \cdot e^x \Big|_0^2 - 2\int_0^2 x \cdot e^x$$

$$= x^2 \cdot e^x \Big|_0^2 - 2\left[x \cdot e^x \Big|_0^2 - \int e^x \cdot dx \right.$$

$$= x^2 \cdot e^x \Big|_0^2 - 2\left[x \cdot e^x \Big|_0^2 - e^x \right]_0^2$$

$$= x^2 \cdot e^x \Big|_0^2 - 2xe^x \Big|_0^2$$

$$= \left[(2)^2 \cdot e^2 - (0)^2 e^0 \right] - \left[2(2)e^2 + 2e^2 - 2(0)e^0 + 2e^0 \right]$$

$$= 4e^2 - 0 - 4e^2 + 2e^2 - 0$$

$$= \cancel{4e^2} - \cancel{4e^2} + 2e^2$$

$$= 2e^2$$

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Q No 3 b) $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Sol #

let $u = \sqrt{x}$

$\Rightarrow du = \frac{1}{2\sqrt{x}} \cdot dx$

$\Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow 2du = \frac{1}{\sqrt{x}}\,dx$

Now

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_1^2 2\sin u \cdot du$$

$$= 2\int_1^2 \sin u \cdot du$$

$$= 2\left[-\cos u\right|_1^2$$

$$= 2\left[-\cos\sqrt{x}\right|_1^2$$

$$= \cancel{2} -\left[2\cos\sqrt{x}\right|_1^2$$

$$= -\left[2\cos\sqrt{2} - 2\cos\sqrt{1}\right]$$

$$= -2\cos\sqrt{2} + 2\cos\sqrt{1}$$

$$= -2\cos\sqrt{2} + 2\cos\sqrt{1}$$

$$= -2(0.99970) + 2(0.9999)$$

$= -1.9994 + 1.99970$

$= 0.0003$ Ans.

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(Q No 4) $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

we saticfy

So that $U_{xx} + U_{yy} + U_{zz} = 0$

$U_x = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$

$U_x = -x(x^2+y^2+z^2)^{-3/2}$

$U_{xx} = -(x^2+y^2+z^2)^{-3/2} - x\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x)$

$U_{xx} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$

$U_{yy} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$

$U_{yy} - y(x^2+y^2+z^2)^{-3/2}$

$U_{yy} = -(x^2+y^2+z^2)^{-3/2} - y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}\cancel{2}y$

$U_{yy} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^3)^{-3/2}$

Similaly.

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$$U_{zz} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$U_{xx} + U_{yy} + U_{zz} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+ 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+ 3z^2(x^2+y^2+z^2)^{-5/3} - (x^2+y^2+z_1)^{-3/2}$$

$$= 3(x^2+y^2+z^2)(x^2+y^2+z^2)^{-5/2} - 3(x^2+y^2+z^2)^{-3/2}$$

$$= 3\cancel{(x^2+y^2+z^2)}^{-3/2} - 3\cancel{(x^2+y^2+z^2)}^{-3/2}$$

$$= 0$$

So laplace eq satisfies-

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