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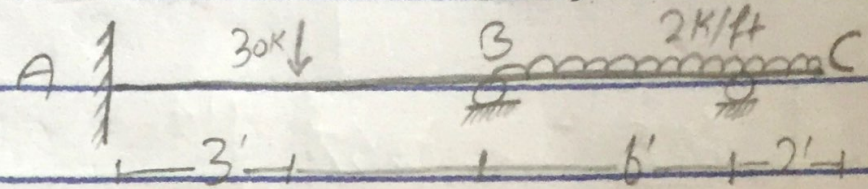
Subject : Structure Analysis II

Submitted to : Engr Adeed.

DATE : 25/Sep/2020

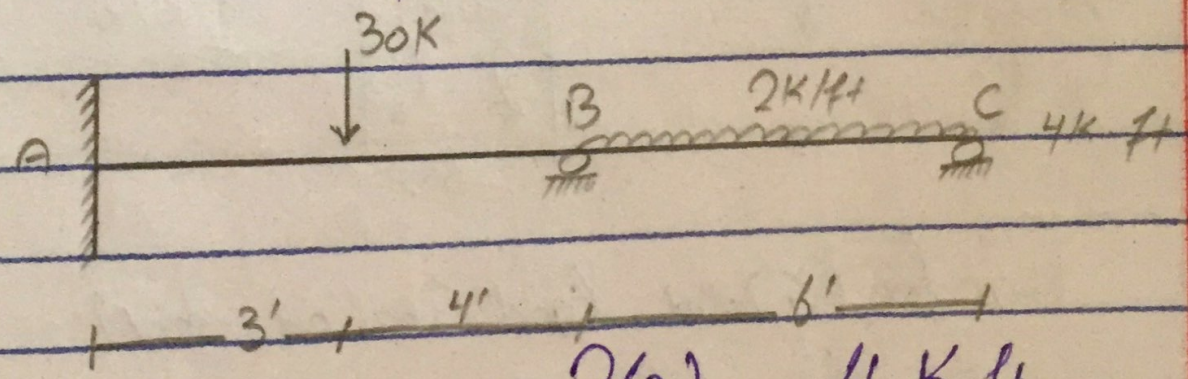
Q#1 Analyze the beam shown in FIG-1 by Stiffness method.
Assume EI is Constant.



Sol:
Step #1
Determine Kinematic Indeterminancy:

K.I = 5°

So we have to reduce the extended Portion.

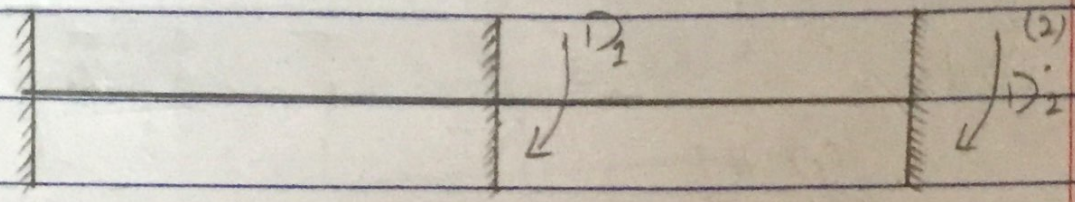


$$\Rightarrow \frac{2(2)}{1} = 4\ K.ft$$

Now:
K.I = 2°

Step #2 :

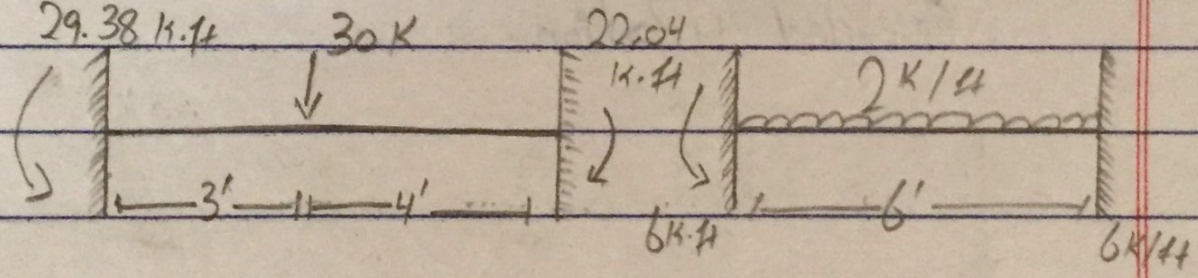
Determine unknown Joint Displacement.



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step #03

Compute [ADL] matrix



→ for Point load (not at mid)

→ for left end:

$$\frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38\ K.ft$$

→ for right end:

$$\frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04\ K.ft$$

⇒ for uniformly distributed load

$$\frac{WL^2}{12} \Rightarrow (2)(6)^2/12 = 6 \text{ K.ft}$$

$$ADL_1 = +22.04 - 6 = 16.04 \text{ K-ft}$$

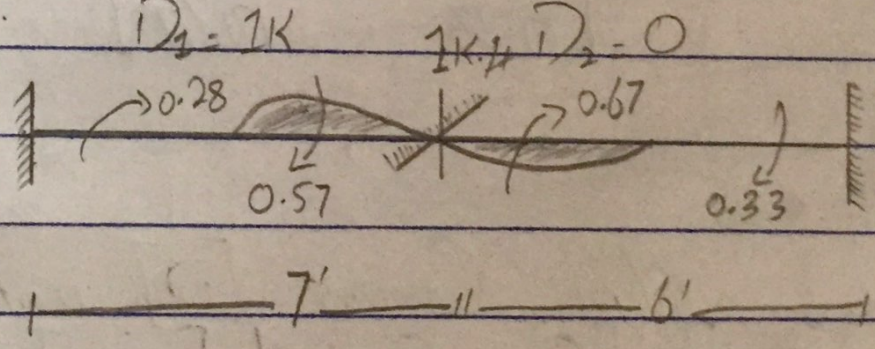
$$ADL_2 = 6 \text{ K.ft}$$

Step # 04

Now Compute [S] Matrix

$$S = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

(a)



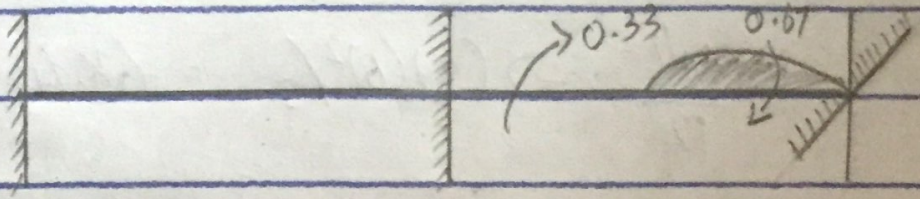
$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$$\delta_{11} = 0.57 + 0.67$$

$$\boxed{\delta_{11} = 1.24EA} \qquad \boxed{\delta_{21} = 0.33EA}$$

b) $D_1 = 0$ $\quad D_2 = 1K$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$\boxed{S_{12} = 0.33} \qquad \boxed{S_{22} = 0.67}$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step #05

Now Compute [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times Adj A \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now

$$\begin{bmatrix} AD_1 - AD_{21} \\ AD_2 - AD_{22} \end{bmatrix} \cdot \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

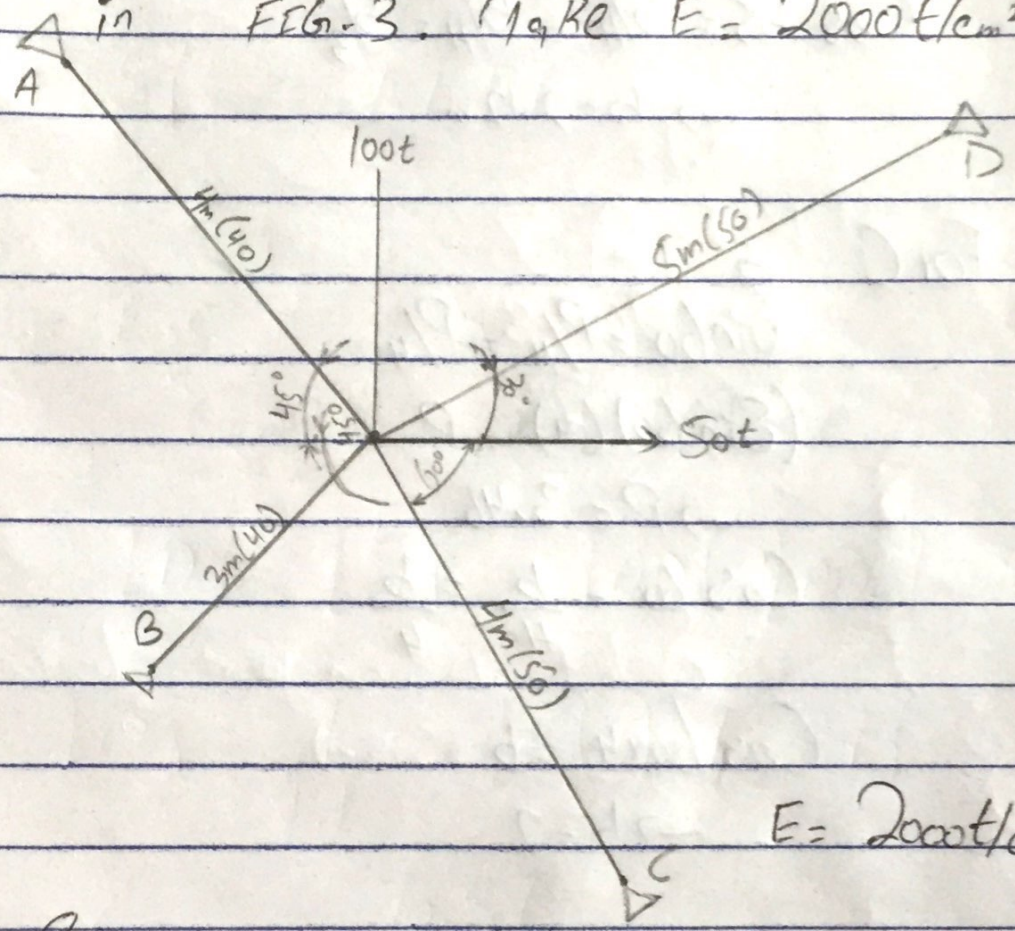
$$\rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{|S|} \times \text{Adj}A \times \begin{bmatrix} -16.04 \\ 2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.729} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.919 & -0.452 \\ -0.452 & 1.70 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.83 \\ 3.85 \end{bmatrix}$$

Q#2 Analyze the Pin-jointed frame shown by Stiffness method. Length of the members in "m" and Cross-Sectional area of the members in Cm^2 are shown in FIG-3. Take $E = 2000 t/cm^2$.



$E = 2000 t/cm^2$.

Sol:-

For A

$$\sin 45 = \frac{P}{H} = \frac{P}{4}$$

$$\Rightarrow P = 2.828 m$$

$$\cos 45° = \frac{b}{H} = \frac{b}{4}$$

$b = 2 = 2.828m$

For B:

$$\sin 45 = \frac{P}{H} = \frac{P}{3}$$

$$\rightarrow P = 2.12m$$

$$\cos 45 = b/H = b/H = b/3$$

$$\rightarrow b = 2.12m$$

For C

$$\sin 60 = P/H = P/4$$

$$(\sin 60)(4) = P$$

$$\rightarrow P = 3.46$$

$$\cos 60 = \frac{b}{H} = \frac{b}{4}$$

$$\cos 60 \times 4 = b$$

$$\rightarrow b = 2$$

For D

$$\sin 30 = P/5$$

$$\rightarrow P = 2.5m$$

$$\cos 30 = b/3$$

$$b = 4.33m$$

Now;

$EA(A) = 2000 \times 40 = 80{,}000\,t$

$EI3(B) = 2000 \times 40 = 80{,}000\,t$

$EA(C) = 2000 \times 50 = 100{,}000\,t$

$EA(D) = 2000 \times 50 = 100{,}000\,t$

Step #01

K.I

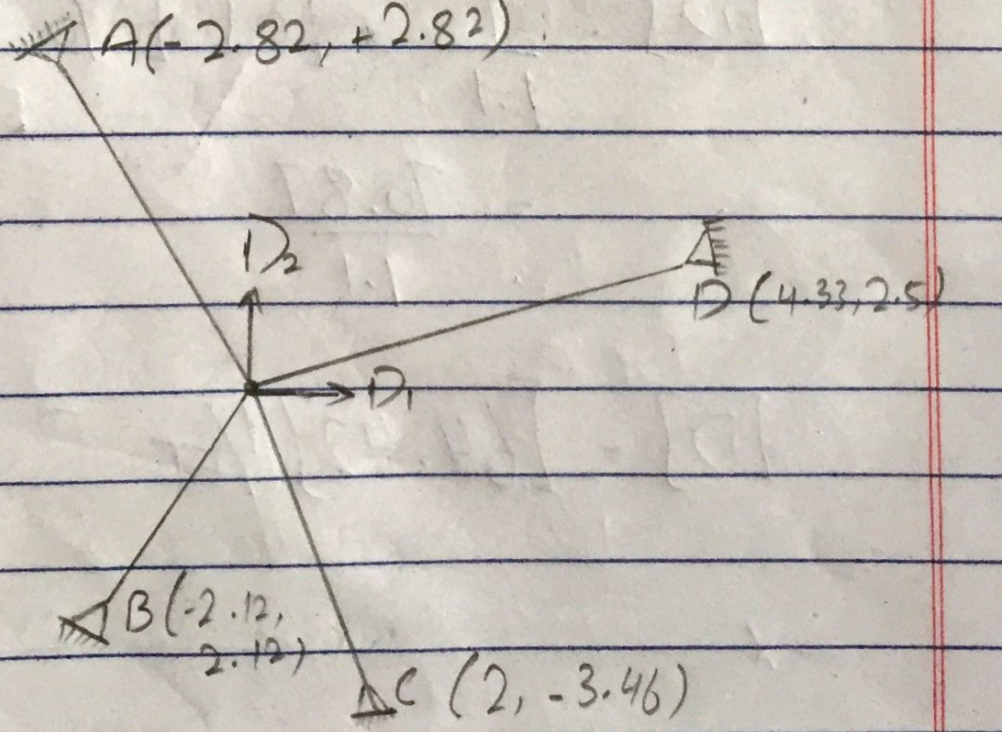
$K.I = 2j - r$

$= 2(5) - 8$

$K.I = 2^{\circ}$

Step #02

Select Unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step #3

$$[AMD]_{4\times 2} \quad [S]_{2\times 2}$$

1) $D_1 = 1K$, $D_2 = 0$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0+282) = 141$$

$$AMD_{31} = \frac{80,000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{21} = \frac{100,000}{(500)^2} \times (0+212) = 188.44$$

$$AMD_{41} = \frac{10,000}{(400)^2} \times (0-200) = -125$$

Now,

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3} (xK - x_j)^2$$

$$= \frac{80,000}{(400)^3} (282)^2 + \frac{80,000}{(300)^3} \times (212)^2$$

$$+ \frac{100,000}{(500)^3} \times (-433)^2 + \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$\Rightarrow S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (x_K - x_j)(y_K - y_j)$$

$$= \frac{80,000}{(400)^3}(282)(-282) + \frac{80,000}{(300)^3}(212)(212)$$

$$+ \frac{100,000}{(500)^3}(.433)(0.250) + \frac{100000}{(400)^3}(-200)$$

$$(0.346)$$

$$\boxed{S_{12} = S_{21} = 17.237}$$

ii) $D_1 = 0 \qquad D_2 = 1K'$

$$AMD = EA/L^2 (y_K - y_j)$$

$$AMD_{12} = \frac{80,000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{80,000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100,000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100,000}{(400)^3}(346) = 216.25$$

Now, $S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(y_K - y_j)^2$

$$= \frac{80,000}{400^3}(-282)^2 + \frac{80,000}{300^3}(212)^2$$

$$+ \frac{100,000}{500^3}(-250)^2 + \frac{100,000}{400^3}(346)^2$$

$$\boxed{\delta_{22} = 469.628}$$

Step #04

$$[D] = [\delta]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \begin{bmatrix} 150 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step #5

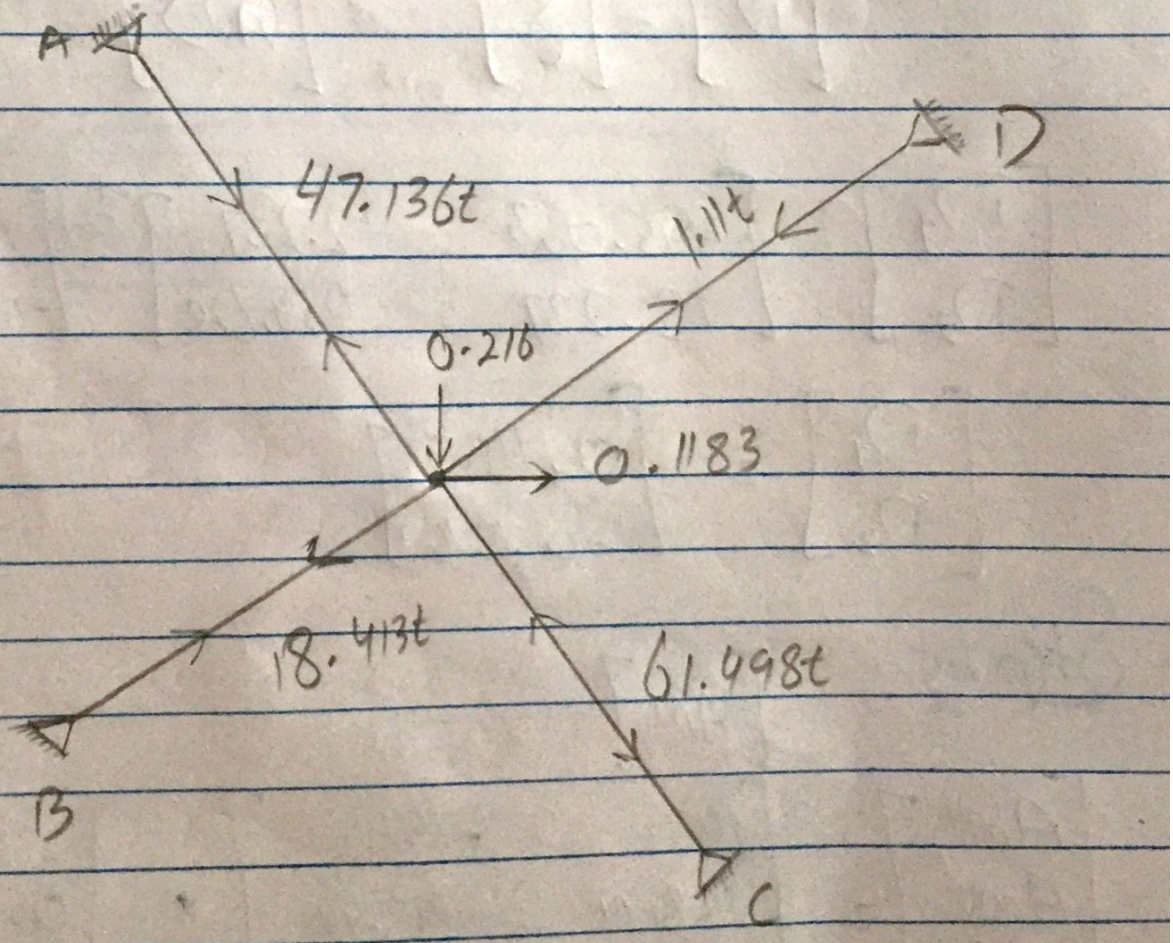
$$[AM]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

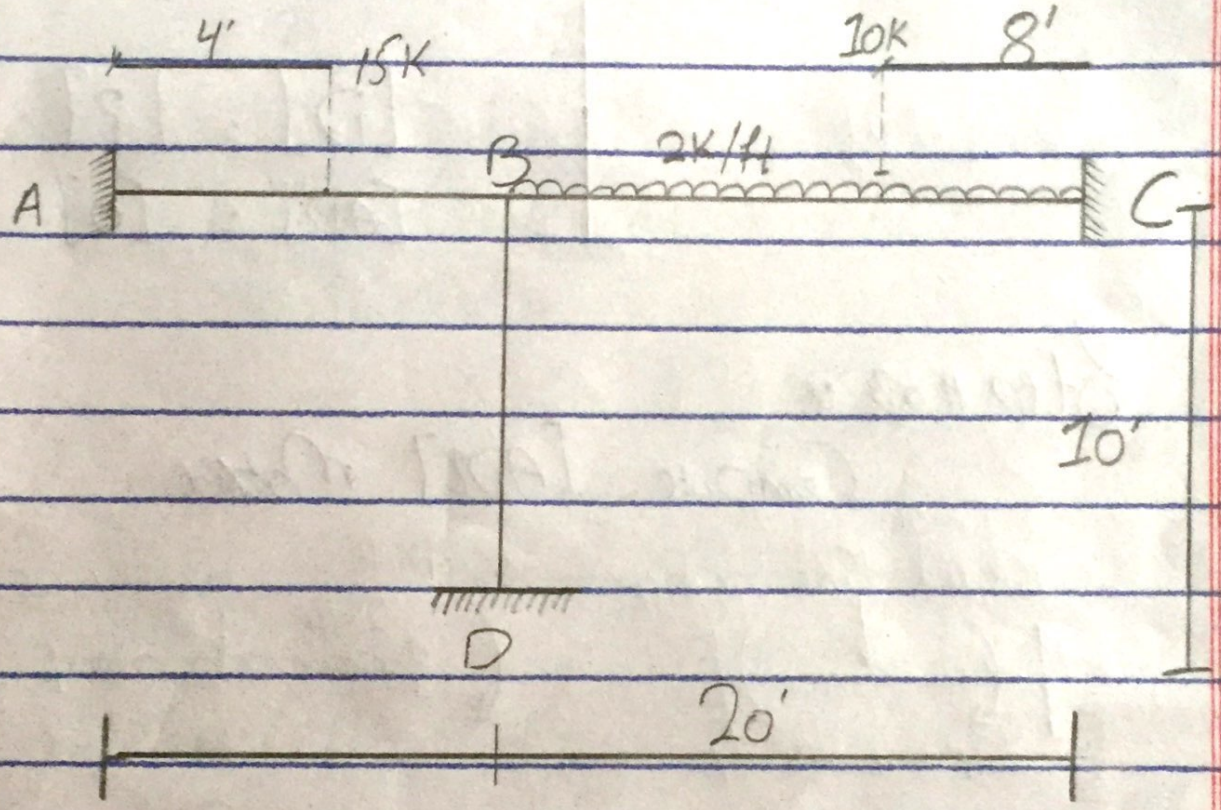
$$\begin{bmatrix} 141 \times 0.1183 + (-141) \times (0.216) \\ 188.44 \times 0.183 + (188.44) \times (-0.216) \\ -173.2 \times 0.1183 + (-100)(-0.216) \\ -125 \times 0.1183 + 216.25 \times (-0.216) \end{bmatrix}$$

$$\begin{bmatrix} Am_1 \\ Am_2 \\ Am_3 \\ Am_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 + 40.70 \\ -20.49 + 21.6 \\ -14.79 + 46.71 \end{bmatrix}$$

$$\begin{bmatrix} Am_1 \\ Am_2 \\ Am_3 \\ Am_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Q#3 Analyze the rigid-joint frame shown in FIG-2 by Stiffness method. Assume EI is Constant.

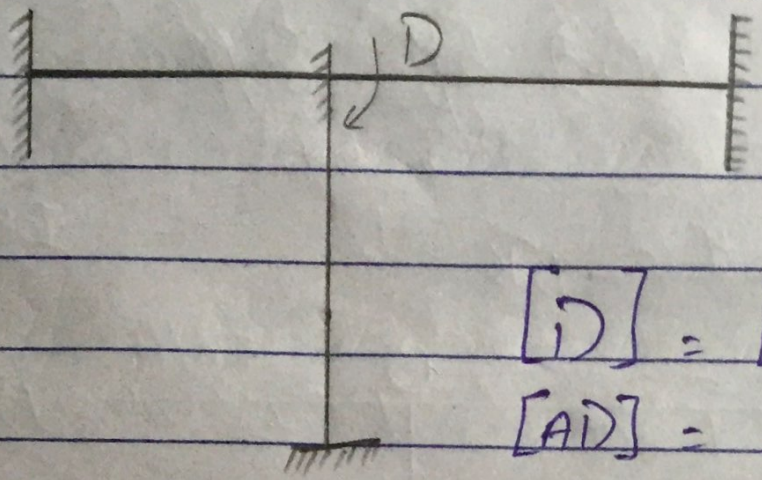


Sol:

Step #01:- Determine Kinematic Indeterminancy.

K.I = 1°

Step #02:- Determine Unknown Joint Displacement.

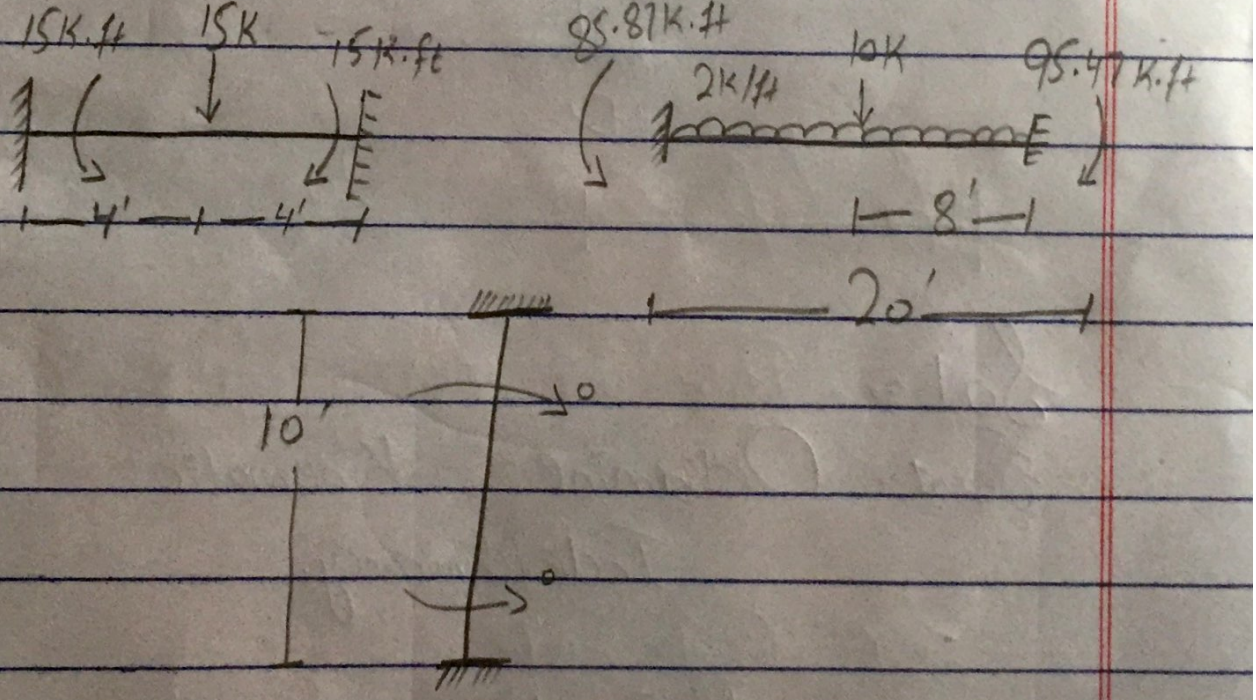


$[D] = [?]$

$[AD] = [0]$

Step #03:

Compute [AD2] Matrix.



=> Point Load at Center:

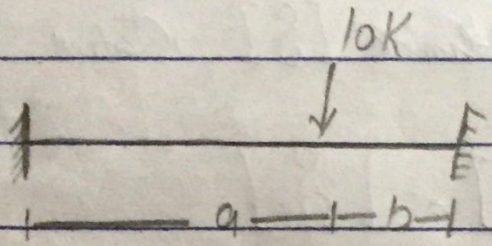
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ Kip-ft}$$

=> Uniformly Distributed Load :

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ K.ft}$$

=> Point load (not at mid) :

Suppose :



for left End :

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ K.ft}$$

for Right End :

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ K.ft}$$

So total moment at left end :

$$19.2 + 66.67 = 85.87 \text{ K.ft}$$

Similarly at Right End:

$$28.8 + 66.67 = 95.47 \text{ K.ft}$$

So $[A_i] = -85.87 + 15 = -70.87$ K.ft

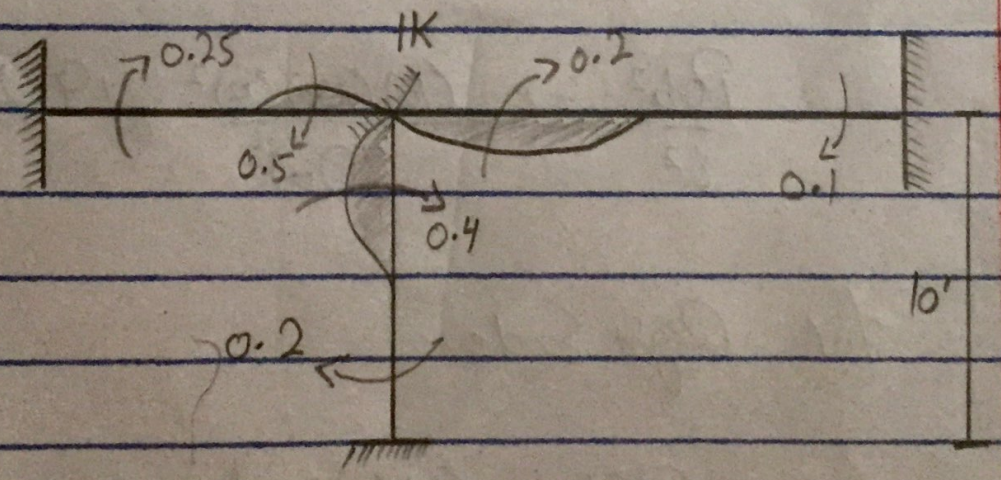
Step #4:

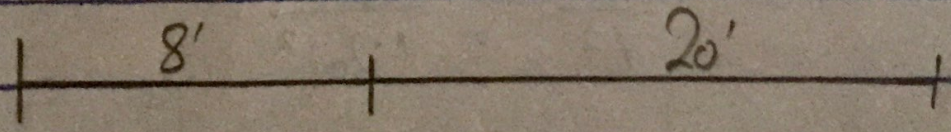
Determine [S] Matrix

$$[S] = [S_{ij}]$$

Now

D = 1K





$$\Rightarrow \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$$

$\Rightarrow \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\ EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

Step # 05

Compute [D] matrix

$[D] = [S]^{-1} \times [AD] - [AiDL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42]\ 1/EI$