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Probability and statistic.

Q1 A MAN throws two fair dice, what is the conditional probability that the sum of the two dice will be 7, given that

1) The Sum is even
2) The Sum is greater than 8
3) The two dice had the Same outcome.

Solution:-

The Sample Space S for this expirment is

$$S = \{ (1,1), (1,2) (1,3)(1,4)(1,5)(1,6)$$
$$(2,1), (2,2) (2,3) (2,4)(2,5)(2,6)$$
$$(3,1), (3,2), (3,3) (3,4)(3,5)(3,6)$$
$$(4,1), (4,2), (4,3) (4,4) (4,5)(4,6)$$
$$(5,1), (5,2), (5,3) (5,4) (5,5) (5,6)$$
$$(6,1), (6,2), (6,3) (6,4) (6,5) (6,6) \}$$

let $A = \{$ the Sum is 7 $\}$

$B = \{$ the Sum is even $\}$

$C = \{$ the Sum is greater than 8 $\}$.

$D = \{$ the two dice had the Same outcomes $\}$

then

$$A = \{(1,1), (1,3)(1,5) (2,2), (2,4)(2,6) (3,1)$$
$$(3,1), (3,3), (3,5) (4,2), (4,4), (4,6)$$
$$(5,1), (5,3) (5,5), (6,2), (6,4), (6,6) \}$$

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$B = \{(3,6), (4,5), (4,6) (5,4), (5,5), (5,6)$
$(6,3), (6,4), (6,5), (6,6)\}$

$C = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$A \cap B = \{(4,6), (5,5), (6,4), (6,6)\}$

$A \cap C = \{(1,1), (2,2), (3,3), (4,4), (5,5)$
$(6,6)\}$

$P(A) = \frac{18}{36}, \; P(B) = \frac{10}{36}, \; P(C) = \frac{6}{36}$

$P(A \cap B) = \frac{4}{36}, \; P(A \cap C) = \frac{6}{36}$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{4}{36} \times \frac{36}{10} = \frac{4}{10} = \frac{2}{5}$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{6}{36} \times \frac{36}{6} = 1$$

(iii)

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Q2 Show that in a single throw of two dice the probability of throwing more than 7 is equal to that of throwing less than, 7 and hence find the probability of exactly 7.

Solution:-

Sum of 2 has 1 way (1,1)
Sum of 3 has 2 ways 1,2 and 2,1
Sum of 4 has 3 ways 1,3 : 2,2 ; 3,1
5 has 4 ways
6 has 5 ways
8 has 5 ways (symmetry)
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 ways

Those are $15/36$ for each side with a sum of $30/36$.

That leaves a $6/36 = 1/6$ probability for a sum of 7.

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Q3 A and B play a game in which A's probability of wining is 2/3. in a Series of 8 games what is the probability that A will win.

1) Exactly 4 games.
2) At least 4 games
3) from 3 to 6 games

Solution:-

Given that $p = \frac{2}{3}$ $n = 8$

$q = 1 - p$

$q = 1 - \frac{2}{3}$

$q = \frac{1}{3}$

let "x" denotes the number of games won by A, then

1) **Exactly 4 games:-**

$P(x=4)$

$= \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$

$= \frac{1120}{6561}$

$= 0.1707$

(ii) **At least 4 games:-**

$P(x \geq 4)$

$1 - P(x<4)$

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$$= 1 - \sum_{x=0}^{3} \binom{8}{x}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right]$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

(iii) <u>from 3 to 6 games:-</u>

$$P(3 \le x \le 6)$$

$$\sum_{x=3}^{6} \binom{8}{x}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

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Q6 Difference between Bi-nominal frequancy distribution and Bi-nominal distribution with the help of formula.

Bi-nominal Frequancy Distribution

Ans "if the bi-nomial probability is multiplied by N, the number of experiments or sets, the resulting distribution is known as the bi-nomial frequancy distribution $N\binom{n}{x}p^x q^{n-x}$"

Bi-nominal distribution:-

Many experiments consist of repeated independents trials, each trials having two possible outcomes for example the two possible outcome of a trial may be head and tail, Succen and failure, light and wrong.

"if the probability of each outcomes remains the Same througout the trials then such trials are called "Bernoulli trials" and the experiment having "n" Bernoulli trials is called "Bionomial experiments".

$$f(x) = {}^{n}C_{x} p^x q^{n-x}$$

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Q7 Below you will find the mean and Standard deviation of several data sets. you're interested in Comparing each data set however, each data set has a different mean Standard deviation and sample size. find the Co-efficient of variation for each data set in the table below.

Measure	Data set A	B	C	D
Mean	45	60	50	25
SD	3	11	5	15
Sample Size	1500	3200	500	2700

Ans.

Solution:-

Measure	Data set A	B	C	D
Co-efficient of variation	$CV = \frac{3}{45} \times 100$ $CV = 6.7$	$CV = \frac{11}{60} \times 100$ $CV = 18.3$	$CV \frac{5}{50} \times 100$ $CV = 10$	$CV = \frac{15}{25} \times 100$ $CV = 60$

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In this case the data set with the lowest c_v is data set A, followed by C, D and D Meaning set A has the lowest variation amongst these data sets.

Q5 Derive the binomial distribution and find its mean and variance.

Ans (i) let "x" be random variable with the bi-nomial distribution $b(x; n, p)$ then its mean is give by

$$E(x) = \mu = np$$

(ii) let "x" be a random variable with the bi-nomial distribution $b(x; n, p)$ then its variance is given by

$$Var(x) = \delta^2 = np.q$$

(iii) let "x" be a random variable with the bi-nomial distribution $b(x; n, p)$, then its standard Deviation is given by

$$\delta = \sqrt{npq}$$

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Q(4) Let $C_1, C_2 \cdots CM C_1, C_2 \cdots CM$ be a partition of the sample space SS, and AA, BB be two events. Suppose we know that.

- A and B are conditionally independent given C_i, for all $i \in (1, 2, \dots M)$
- B is independent of all C_i.s.

Prove that A and B are independent.

Two event A and B are condionally independent given an event C with $P(C) > 0$ if

$$P(A \cap B | C)$$
$$= P(A|C)\, P(B|C) \rightarrow (a)$$

Recall that from the difinition of conditional probability.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

if $P(B) > 0$. By conditioning on C, we obtain

$$P(A|B, C) = \frac{P(A \cap B|C)}{P(B|C)}$$

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if $P(B|C), P(C) \neq 0$. if A and B are conditionally independent given C, we obtain

$$P(A|B,C) = \frac{P(A \cap B|C)}{P(B|C)}$$

$$= \frac{P(A|C)\,P(B|C)}{P(B|C)}$$

$$= P(A|C)$$

Thus if A and B are conditionally independent given C, then

$$P(A|B,C) = P(A|C) \quad \text{—(b)}$$

Thus equation (a) and (b) are equivalent statements of the definition of conditional independence.

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