Q1) $f(t) = 1+t, \quad -\pi \le t \le \pi$

Sol:-

Here we use the formula

$$f(u) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad -(i) \text{ eq}(1)$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\,dt$$

$$\Rightarrow a_0 = 1/2\pi \int_{-\pi}^{\pi} (1+t)\,dt$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right)$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(2\pi + \frac{\cancel{2}\pi^2}{\cancel{2}} \right)$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)$$

$$\Rightarrow a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\,dt$$

$$\Rightarrow a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt}{n} \frac{d}{dt}(1+t) \right) \right)$$

$$\Rightarrow a_n = \frac{1}{\pi} \left(\cancel{(1+t) \frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$\Rightarrow a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$\Rightarrow a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\Rightarrow \boxed{a_n = 0}$$

*) $$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\, dt$$

$$\Rightarrow b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt - \frac{d}{dt}(1+t)\,dt\right)\right.$$

$$\Rightarrow b_n = \frac{1}{\pi}\left(\left.\frac{(1+t)(-\cos nt)}{n}\right)\right]_{-\pi}^{\pi} - \int\left(-\frac{\cos nt}{n}(1)\right)$$

$$\Rightarrow b_n = \frac{1}{\pi}\left(-\left.\frac{(1+t)(\cos nt)}{n}\right]_{-\pi}^{\pi} + \left(\left.\frac{\sin n u}{n^2}\right]_{-\pi}^{\pi}\right)\right)$$

$$\Rightarrow b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - \left((1+(\pi))(\cos nt)\right)\right)$$

$$\Rightarrow b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right)$$

$$\Rightarrow b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \dfrac{(-1)^{n+1}}{n}$

$\rightarrow \boxed{b_n = \frac{2}{n} (-1)^{n+1}}$

so equation becomes

$$f(x) = \frac{1}{2\pi}(2\pi + \pi^2) + 0 + \sum_{n=1}^{\infty} \frac{2 \cdot (-1)^{n+1}}{n} \sin t.$$

x ——— x ——— x ——— x ——— x

Q2) $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

Eigen values?

Sol:-

Step # 1:-

we have;

$(A - \lambda I)x = 0$ A = Given matrix

Step # 02 :-

we have; The characteristics equation is given by;

$(A - \lambda I) = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step # 3 :-

$$\lambda^3 - \left|\begin{matrix}\text{sum of} \\ \text{diagonal element}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{sum of} \\ \text{diagonal minor}\end{matrix}\right| \lambda - |A| = 0 \quad \text{---(B)}$$

Sum of diagonal elements $= 1+1+2 = 4$

Sum of diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$\boxed{= -3}$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{---(C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$\boxed{= 0}$

By putting values in (c);

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

using quadratic formula:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -4$
$c = -3$

$$\lambda = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$\lambda = \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

we have eigen values;

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$ Required Solution.

Q3) Solve the following System of linear equations:-

$$5x + 0 + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z + 0 = 1$$
$$x + y + z + m = 0$$

Sol:-

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] R_4 \quad R_2$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+4/5 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] -1/5 \times R_3$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \underline{5 \times R_3} \text{ and } \underline{5 \times R_4}$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \text{ and } 5R_4$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \longleftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad C_2 \times -5$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(u, y, z, m) = (3/4,\ 31/21,\ -11/21,\ 1/3)$$

$u = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$

Q4) verify that $u(x,t) = \sin(x+2t)$ is a solution of the one dimensional equation.

Sol:

Given that

$u(x,t) = \sin(x+2t)$

Differentiate w.r.t x partially

$$\Rightarrow \frac{du}{dx} = \frac{d}{dx} \sin(x+2t)$$

$$\rightarrow \frac{du}{dx} = \cos(x+2t) \frac{\partial}{\partial x}(x+2t)$$

$$\rightarrow \frac{du}{dx} = \cos(x+2t)(1+0)$$

$$\Rightarrow \frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x+2t)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \frac{\partial}{dx}(x+2t)$$

$$\Rightarrow \frac{d^2u}{du^2} = -\sin(u+2t)(1+0)$$

$$\Rightarrow \boxed{\frac{d^2u}{du^2} = -\sin(u+2t)}$$

and $u(u,t) = \sin(u+2t)$

Differentiat w.r.t "t".

$$\Rightarrow \frac{du}{dt} = \frac{d}{dt}\sin(u+2t)$$

$$\Rightarrow \frac{du}{dt} = \cos(u+2t)(0+2)$$

$$\Rightarrow \frac{du}{dt} = 2\cos(u+2t)$$

$$\Rightarrow \frac{d^2u}{dt^2} = (2)-\sin(u+2t)(0+2)$$

$$\Rightarrow \boxed{\frac{d^2u}{dt^2} = -4\sin(u+2t)}$$

As we know that one dimensional wave equation is :

$$\frac{d^2u}{dt^2} = c^2 \frac{d^2u}{dx^2}$$

$$-4\sin(x+2t) = c^2 [-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

for the orbitary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the orbitary constant.

$$c = 2.$$