Name: Faham Akhtar

ID No:- 15772

Assignement:- Linear Algebra

Teacher :- Mansoor Qadir.

Dept:- Bs (412).

- The linear independence property for every finite subset $\{b_1, \dots b_n\}$, of B and every $a_1 \dots a_n$ in F if $a_1 b_1 + \dots + a_n b_n = 0$ then $a_1 = \dots = a_n = 0$;

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Ans

Ans

Theorem 1: The Subspace Spanned by a non-empty Subset S of a vector space V is the the Set of all linear Combination of vector in S. This theorm is So well Known that all times it is referred to as the definition of span of a Set

(4) Ans In mathematices, the dimension of a vector space V is the Cardinally of a basis of V over its base field. it is sometimes called

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Ans

For a homogenous system of equation $ax + by = 0$ and $cx + dy = 0$, the situation is slightly different.. A $n \times m$ homogoues system of linear equation has a unique solution (the trival solution) if ~~it~~ and only if its determinant it zero. then the system has an infinite number of solution.

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Ans

General solution to a Nonhomogeneous linear equation

A equation $yp(x)$ of a different equation that contains no arbitary contants is called a Paticular solution to the equation.

$a_2(x)y'' + a_1(x)y' + a_0(x)y = r(x)$

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Ans

A System is called Consistent if it has a Solution. A General Solution of a System of linear equation is a formula which gives all Solution for different values paraments. This System has just one Solution $x=5, y=2$

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Ans

Let all be vector Subspace of the vector Space. we define direct Sum. of those subspace. \oplus U-m = V$ is defined to be a Sum of the Subspace to which each element in can be uniquely written as.--
Lemma 1 : Let be ve[illegible] Subspace of the vector space.

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Ans

The orthogonal complement is a subspace of vector where all of the vectors in the orthogonal to all of the vector in a particular subspace for instance, if you are given a plane in R^3, then the orthogonal complement of the plane and that passes throught.

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matrix is a Polynomial which is invariant under matrix Similarity and has the eigenvalues as roots. The Characteristic equation is the obtained by equating to zero the Characteristic Polynomial.

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~~The orthogonal Complement is a Subspace of vector where all of the vectors in the~~

Ans

In mathematics, an equivalance is a baniry relation "is equal to" is the canonical example of an equivalance relation, where for any objects a, b, and c: a = a = b then b = a and if a = b and b = c than

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Ans

The kernal or null-space of a linear transformation is the set of all the vectors of the input space that are mapped under the linear transformation to the null vector of output space.

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Ans

Let $T: R^5 \to R^7$ be a linear transformation

. Find the dimention of the kernel of T if the demions of the range is 2

$\dim(\text{kernel}) = n - \dim(\text{range})$

$= 5 - 2 = 3$

. Find the rank of T if the nullity of T is 4

$\text{rank}(T) = n - \text{nullity}(T) = 5 - 4 = 1$

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Ans

The Image of a linear transformation or matrix is the Span of a Vector of the what vector you can get from applying the linear transformation or multiplying the matrix by a vector.). it can written as Im(A).

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Ans

The rank of a linear transformation L is the dimension of its Image written Rank L.
1. The nullity of a linear transformatio is the dimension of the kernal, written L. Theorem (Dimension form Let L : V → W be a linear trasformations with V a finite dimenisonal vector space2.

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Ans

In linear algebra. the Characteristic Polynomial of a Squ

Signature

Hamel dimension or algebraic dimension to distinguish it from other types of dimension

5

Ans

In linear algebra, an eigenvector ('/ aɪɡən.vɛktər/) or Charateristic vector of a linear transformation is a nonzero vector that changes at most by a Scalar factor when that linear transformation is applied to it The corresponding eigenvalue is the factor by which the eigenvector is Scaled.

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Ans

. In mathematics and more Specifically in linear algebra. a linear subspace, also know as a vector subspace, also known as a vector space is a vector space that is a Subset of Some larger vector space.

3:- The Characteristic frequences are $0, \omega$

4:- The orthogonal change of basis matrix.

$$P = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix}.$$

Kings Notes

Date:

It obeys $MP = PD$ where

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix}.$$

5:- Yes, the Direction given by eigen vector.

$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ because it's eigenva

if zero. This is probably a bad design ~~too~~ for a bridge because it can be displaced in this direction with no force.

The eigenvectors, when $\lambda=0$ we

$$M-0\cdot I=\begin{pmatrix}-1 & -1 & 0\\ -1 & -2 & -1\\ 0 & -1 & -1\end{pmatrix}\sim\begin{pmatrix}1 & 1 & 0\\ 0 & -1 & -1\\ 0 & -1 & -1\end{pmatrix}\sim\begin{pmatrix}1 & 0 & -1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{pmatrix}$$

So $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ is an eigenvector. For $\lambda=-1$.

$$M(-1)\cdot I=\begin{pmatrix}0 & -1 & 0\\ -1 & -1 & -1\\ 0 & -1 & 0\end{pmatrix}\sim\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

So $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ is an eigenvector. For $\lambda=-3$

$$M-(-3)\cdot I=\begin{pmatrix}2 & -1 & 0\\ -1 & 1 & -1\\ 0 & -1 & 2\end{pmatrix}\sim\begin{pmatrix}1 & -1 & 1\\ 0 & 1 & -2\\ 0 & -1 & 2\end{pmatrix}$$

So $\begin{pmatrix}1\\2\\1\end{pmatrix}$ is an eigenvector.

3:- The characteristic frequences are 0, 1

4:- The orthogonal change of basis matrix.

$$P=\begin{pmatrix}1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6}\\ -1/\sqrt{3} & 0 & 2/\sqrt{6}\\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6}\end{pmatrix}$$

Kings Notes

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Q4:- A team of distinguished post-doctoral engineers analyzes the design for a bridge ... by an amount.

Answer:-

$$\frac{d^2x}{dx^2} = \frac{d^2 \cos(\omega t)}{dt^2}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$= -\omega^2 \cos(\omega t)\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Hence

$$F = \cos(\omega t)\begin{pmatrix} -a-b \\ -a-2b-c \\ -b-c \end{pmatrix} = \cos(\omega t)\begin{pmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$= -\omega^2 \cos(\omega t)\begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

2:- So

$$M = \begin{pmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

$$\det\begin{pmatrix} \lambda+1 & 1 & 0 \\ 1 & \lambda+2 & 1 \\ 0 & 1 & \lambda+1 \end{pmatrix} = (\lambda+1)((\lambda+2)(\lambda+1)-1)-(\lambda+1)$$

$$= (\lambda+1)(\lambda^2+3\lambda) = \lambda(\lambda+1)(\lambda+3)$$

So the eigen values are, ~~$\lambda=0, \lambda=1$~~ $= \lambda = 0, -1, -3$ for the eigenvectors, when $\lambda = 0$ we study.

Kings Notes

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The solution set is $\left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -\mu \\ \mu \end{pmatrix} : \mu \in R \right\}$

$\Longleftrightarrow$

Q No: 3:- Compute the following determin

Answer:- $\text{Det} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = -2.$

All the other determinants vanish because the first three rows of each matrix are not independent. Indeed, $2R_2 - R_1 = R_3$ in each case, so we can make row operations to get a row of zeroes, and thus a zero determinant.

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Forward substitution
$a=1, a+b=1, a+b+c=1 \Rightarrow a=1, b=0, c=0$

Now solve $Ux=w$ by back substitution
$x+y+z+w=1, y+z+w=0, z+w=0.$

$\Rightarrow w=\mu$ (arbitrary), $z=-\mu, y=0, x=1.$

~~Then~~
Then $M=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{pmatrix}.$

$=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}=LU.$

So now $Mx=v$ becomes $Lw=v$
where $w=Ux=\begin{pmatrix} a \\ b \\ c \end{pmatrix}$. Thus

we solve $Lw=v$ by forward substitution
$a=1, a+b=1, a+b+c=1 \Rightarrow a=1, b=0, c=0$
Now solve $Ux=w$ by back substit
$x+y+z+w=1, y+z+w=0, z+w=0$

$\Rightarrow w=\mu$ (arbitrary), $z=-\mu, y=0, x=1.$

Q2:- Consider the system of equation

$$x+y+z+w=1$$

$$x+2y+2z+2w=1$$

$$x+2y+3z+3w=1$$

Express this system as a matrix equation $Mx=V$ and then find the solution by computing an LU decomposition of the Matrix M?

Answer:-

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Then

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

So now $Mx=V$ becomes $LW=V$ where $W = Ux = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ Thus we solve $LW = V$ by