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"Signal and system"

Q1 (a) We know that differentiation in time domain corresponds to multiplication by $j\omega$ in frequency domain from the duality property. we might suspect that multiplication by jt in the time domain corresponds roughly to differentiation in frequency domain.

we know that.

~~Differentiating bs wrt $j\omega$~~

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

Diff both sides wrt ω.

$$\frac{d}{d\omega} x(j\omega) = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-j\omega t}\, dt$$

$$\frac{d}{d\omega} x(j\omega) = -jt \int_{-\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\frac{dx}{d\omega} j(\omega) = -jt\ \mathcal{F}\{x(t)\}.$$

$$-jt\,(x(t)) \xleftrightarrow{\mathcal{F}} \frac{d}{dt} x(j\omega)$$

1 Q b if $x(n) = 2\delta(n) - 4\delta[n-2] + 2\delta(n-3]$

$h[n] = 3\delta(n) + \delta(n-1] + 2\delta(n-2)$

Produce $Y(z)$ and $y(n)$.

Sol:

$$X(z) = 2 - 4z^{-2} + 2z^{-3}$$

$$H(z) = 3 + z^{-1} + 2z^{-2}$$

Now $Y(z) = H(z) \times X(z)$.

$$= (2 - 4z^{-2} + 2z^{-3})(3 + z^{-1} + 2z^{-2}).$$

$$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} + 2z^{-4} + 4z^{-5}$$

$$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$$

To find $y[n]$ use the delay Property.

$$y[n] = 6\delta(n) + 2\delta[n-1] - 8\delta[n-2) + 2\delta[n-3) - 6\delta[n-4] + 4\delta[n-5].$$

Q2 $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$

Retrieve the fourier series of the given function:

Sol: fourier Series of the Cooffecient a_0.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \left(\int_{-\pi}^{0} -\frac{\pi}{2}\, dx + \int_{0}^{\pi} \frac{\pi}{2}\, dx \right).$$

$$= \frac{1}{\pi} \left(-\frac{\pi}{2} \int_{-\pi}^{0} 1\, dx + \frac{\pi}{2} \int_{0}^{\pi} 1\, dx \right).$$

$$= \frac{1}{\pi} \left\{ -\frac{\pi}{2} (x) \Big|_{-\pi}^{0} + \frac{\pi}{2} (x) \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{\pi} \left[-\frac{\pi}{2} (0) - (-\pi) \right] + \frac{\pi}{2} (\pi) - (0).$$

$$= \frac{1}{\pi} \left(-\frac{\pi^2}{2} + \frac{\pi^2}{2} \right.$$

$$= \frac{1}{\pi} (0)$$

$$\boxed{a_0 = 0}$$

for Coffeicent a_n

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -\frac{\pi}{2} \cos nx \, dx + \int_{0}^{\pi} \frac{\pi}{2} \cos nx \, dx$$

$$= \frac{1}{\pi} \left[-\frac{\pi}{2} \int_{-\pi}^{0} \cos nx \, dx + \frac{\pi}{2} \int_{0}^{\pi} \cos nx \, dx \right.$$

$$= \frac{1}{n\pi} \left[-\frac{\pi}{2} (\sin nx) \Big|_{-\pi}^{0} + \frac{\pi}{2} (\sin nx) \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{n\pi} \left[-\frac{\pi}{2} (\sin n(0) - \sin n(-\pi)) + \frac{\pi}{2} (\sin n(\pi) - \sin n(0)) \right.$$

$$= \frac{1}{n\pi} \left(-\frac{\pi}{2} (0) + \frac{\pi}{2} (0) \right] .$$

$$= \frac{1}{\pi \pi} (0) = 0$$

$$\boxed{a_n = 0}$$

Now for the b_n.

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx.$$

$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} -\frac{\pi}{2} \sin nx \, dx + \int_{0}^{\pi} \frac{\pi}{2} \sin nx \, dx \right.$$

$$= \frac{1}{\pi} \left(-\frac{\pi}{2} (-\cos nx \Big|_{-\pi}^{0} + \frac{\pi}{2} (-\cos nx \Big|_{0}^{\pi} \right.$$

$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left(-\cos n(0) - (-\cos n(-\pi)\right) + \frac{\pi}{2}\left(-\cos n(\pi)\right.\right.$

$\left.\left. - (-\cos n(0)\right)\right].$

$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(-2) + 2\frac{\pi}{2}\right].$

$= \frac{1}{n\pi}\left(2\frac{\pi}{2} + 2\frac{\pi}{2}\right).$

$= \frac{1}{n\cancel{\pi}}\left[\frac{\overset{2}{\cancel{4}}\cancel{\pi}}{\cancel{2}}\right]. \Rightarrow \frac{1}{2n}$

$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{1}{2n} & \text{if } n \text{ is odd.} \end{cases}$

$= 0 + 0 + 0 + \cdots + \frac{1}{2}\sin x + \frac{1}{4}\sin 2x + \frac{1}{6}\sin 3x \cdots$

$\cdots\cdots$

Q3 If $X(z) = \frac{2z^2 + 2z}{(z^2+2z-3)}$

Inverse z-transform:

Sol:

$$x(z) = \frac{2z^2 + 2z}{z^2+2z-3}$$

$$x(z) = \frac{2z(z+1)}{z^2+2z-3}$$

$$\frac{x(z)}{z} = \frac{2(z+1)}{z^2+3z-z-3} = \frac{2(z+1)}{z(z+3)-1(z+3)}$$

$$= \frac{2(z+1)}{(z+3)(z-1)}$$

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{A}{z+3} + \frac{B}{z-1} \longrightarrow \text{i}$$

$$2(z+1) = A(z-1) + B(z+3). \longrightarrow \text{ii}$$

Put $z = 1$ in ii

$$2(1+1) = A(1-1) + B(1+3)$$

$$4 = 0 + B(4). \quad 4B = 4$$

$$\boxed{B = 1}$$

Now Put $Z = -3$ in eq (ii) (7)

$$2(-3+1) = A(-3-1) + B(-3+3).$$

$$-4 = -4A$$

$$\boxed{A = 1}$$

Now to Put A and B in eq (i)

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

Inverse z-transform

$$x[n] = U[3] + 1(-1]^{k}$$

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Q4 Transfer function:

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix},\ B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\ C = \begin{bmatrix} 1 & 2 \end{bmatrix}\ D = [0].$$

Sol: $G(s) = C(sI-A)^{-1}B + D$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{pmatrix} s+2 & 1 \\ 0-1 & s-0 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + [0].$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix}\frac{1}{s^2+2s+1}\begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1}\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{pmatrix} s \\ 1 \end{pmatrix}$$

$$= \frac{1}{s^2+2s+1}\begin{bmatrix} s & 2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s & 2 \end{bmatrix}}{s^2+2s+1}$$

Q5 <u>fourier transform:</u>

$$x(t) = e^{-a|t|}\, u(t).$$

Sol: The fourier transform of the given function $x(t)$ is given by

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

$e^{-a|t|} = e^{-at}$ for $x \geq 0$

$$x(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$e^{-a(-t)} = e^{at}$ for $x < 0$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Big|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Big|_{0}^{\infty}$$

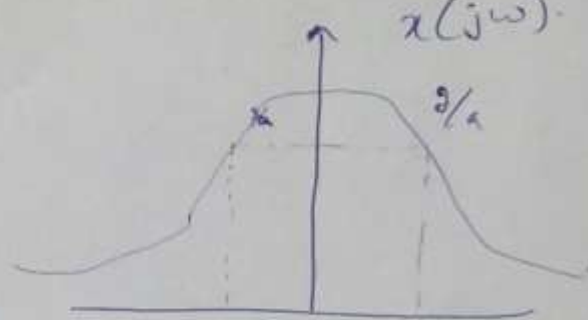
$$= \frac{1}{(a-j\omega)}\left(e^{0} - e^{-\infty}\right) \frac{-1}{(a+j\omega)}\left(e^{-\infty} - e^{0}\right).$$

$$= \frac{1}{(a-j\omega)}\left(1-0\right) - \frac{1}{(a+j\omega)}\left(0-1\right).$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a + \cancel{j\omega} + a - \cancel{j\omega}}{a^2 - j\omega^2}$$

$$= \frac{2a}{a^2 + \omega^2}$$



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