Q1 (a) Define 2nd order linear homogenous/ non-homogenous differential equation along with examples?

Ans Homogenous: that differential equation of any order is homogenous if once all the terms involving the unknown function are collected together an one side of the equation. & the other side is identically zero

Exp

$$y' - 2y' + y = 0$$

NON homogenous

the non-homogenous differential equation has terms on both sides in this type of equation has the form of

$$y'' + py' + qy = f(x)$$

where p.q are real no & can be real & complex.

Azaz Ali 11440 (2)

Q1 (b) (1)

(1) $4y'' - 6y' + 7y = 0$

$$7y(x) - 6\frac{d}{dx}y(x) + 4\frac{d^2}{dx^2}y(x) = 6$$

Sol 4

weged the equation

$$\frac{7y(x)}{4} - \frac{3\frac{d}{dx}y(x)}{2} + \frac{d^2}{dx^2}y(x) = 0$$

$y'' + p*y + q*y = 0.$

where $p = -\frac{3}{2}$

$q = \frac{7}{4}$

$y'' + p*y' + q*y = 0$

$q + (k^2 + kp) = 0$

$k^2 - \frac{3k}{2} + \frac{7}{4} = 0$

$k_1 = \frac{3}{4} - \frac{\sqrt{19}i}{4}$

$k_2 = \frac{3}{4} + \frac{\sqrt{19}i}{4}$

$y(x) = e^{k_1 x}C_1 + e^{k_2 x}C_2$

the final answer

$$y(x) = C_1 e^{x\left(\frac{3}{4} - \frac{\sqrt{19}i}{4}\right)} + C_2 e^{x\left(\frac{3}{4} + \frac{\sqrt{19}i}{4}\right)}$$

Q1 (b) (ii)

$$y''-4y'-12y=3e^{(5x)}$$

Sol Given the equation.

$$-12y(x)-4\frac{d}{dx}y(x)+\frac{d^2}{dx^2}y(x)=3e^{5x}$$

this differential equation has the form

$$y''+p^*y'+q^*y=s.$$

where $p=-4$

$q=-12$

$s=-3e^{5x}$

$$y''+p^*y'+q^*y=0$$

First of all we should find the roots of the characteristic equation

$$q+(k^2+kp)=0$$

in the case the characteristic equation will

$$k^2-4k-12=0$$

then is a simple quadratic equation the roots of the equation

$k_1=2$

$k_2=6$

$$y''+p^*y'+q^*y=s$$

use variation of parameters method suppose th $c_1$ and $c_2$. it is function of $x$

the genral solution is

$$y(x)=c_1(x)e^{-2x}+c_2(x)e^{6x}$$

where $c_1(x)$ and $c_2(x)$

by the method of variation of parameters

we find the solution from the system

$$y_1(x)\frac{d}{dx}C_1(x)+y_2(x)\frac{d}{dx}C_2(x)=0$$

$$\frac{d}{dx}c_1(x)\frac{d}{dx}y_1(x)+\frac{d}{dx}c_2(x)\frac{d}{dx}y_2(x)=$$

where

$y_1(x)$ and $y_2(x)$ - linearly independent particular solutions of Linear Ordinary Differential equation.

$y_1(x)=\exp(-2*x)\ (C_1=1, C_2=0)$,

$y_2(x)=\exp(6*x)\ (C_1=0, C_2=1)$

The free term $f=-5$, or

$$k_2=6$$

$$y(x)=C_1e^{k_1x}+C_2e^{k_2x}$$

$$y(x)=C_1e^{-2x}+C_2e^{6x}$$

$$y''+p*y'+q*y=s$$

use variation of parameters method suppose that $C_1$ and $C_2$ it is function of $x$

The general solution is

$$y(x)=C_1(x)e^{-2x}+C_2(x)e^{6x}$$

wher $C_1(x)$ and $C_2(x)$

by the method of variation of parameters

we find the solution from the system.

$$\frac{d}{dx}c_1(x)\frac{d}{dx}y_1(x)+\frac{d}{dx}c_2(x)\frac{d}{dx}y_2(x)$$

where

$y1(x)$ and $y2(x)$ - linearly independent particular solutions of linear Ordinary Differential equations

$y1(x) = exp(-2*x)$ $(C1=1, C2=0)$.

$y2(x) = exp(6*x)$ $(C1=0, C2=1)$.

The free term $f = -5$, or

$$f(x) = 3e^{5x}$$

So the System has the form

$$e^{6x}\frac{d}{dx}C2(x) + e^{-2x}\frac{d}{dx}C1(x) = 0$$

$$\frac{d}{dx}C1(x)\frac{d}{dx}e^{-2x} + \frac{d}{dx}C2(x)\frac{d}{dx}e^{6x} = 3e^{5x}$$

or

$$e^{6x}\frac{d}{dx}C2(x) + e^{-2x}\frac{d}{dx}C1(x) = 0$$

$$f(x) = 3e^{5x}$$

So the system has the form

$$e^{6x}\frac{d}{dx}C2(x) + e^{-2x}\frac{d}{dx}C1(x) = 0$$

$$\frac{d}{dx}C1(x)\frac{d}{dx}e^{-2x} + \frac{d}{dx}C2(x)\frac{d}{dx}e^{6x} = 3e^{5x}$$

or

$$e^{6x}\frac{d}{dx}C2(x) + e^{-2x}\frac{d}{dx}C1(x) = 0$$

$$6e^{6x}\frac{d}{dx}C2(x) - 2e^{-2x}\frac{d}{dx}C1(x) = 3e^{5x}$$

Solve the system.

$$\frac{d}{dx}C1(x) = \frac{-3e^{7x}}{8}$$

$$\frac{d}{dx}C1(x) - \frac{-3e^{-x}}{8}$$

or

$$e^{6x}\frac{d}{dx}C2(x) + e^{-2x}\frac{d}{dx}C1(x) = 0$$

$$6e^{6x}\frac{d}{dx}C2(x) - 2e^{-2x}\frac{d}{dx}C1(x) = 3e^{5x}$$

Solve the System

$$\frac{d}{dx} C2(x) = \frac{3e^{-x}}{8}$$

it is the simple differential equation solve these equation

$$C1(x) = C3 + \int \left(-\frac{3e^{7x}}{8}\right) dx$$

$$C2(x) = C4 + \int \frac{3e^{-x}}{8} dx$$

$$\cancel{\frac{d}{dx} C2(x)\ \frac{3e^{-x}}{}}$$

or

$$C1(x) = C3 - \frac{3e^{7x}}{56}$$

$$C2(x) = C4 - \frac{3e^{-x}}{8}$$

Substitute found $C1(x)$ and $C2(x)$ to

$$y(x) = C1(x)e^{-2x} + C2(x)e^{6x}$$

the final answer

$$y(x) = C3e^{-2x} + C4e^{6x} - \frac{3e^{5x}}{7}$$

where C3 and C4 is a constants

Azaz Ali 11440 (7)

Q2 (1) $16y' - 40y' + 25y = 0 \quad y(0) = 3 \; y'(0) = -9/4$

sol

Given equation

$16y'' - 40y' + 25y = 0$

$\div$ing whole equation by 16

$$\frac{25y(x)}{16} - \frac{5\frac{d}{dx}y(x)}{2} + \frac{d^2}{dx^2}y(x) = 0$$

where $p = \frac{-5}{2}$

$q = \frac{25}{16}$

To find Roots

$= q + (k^2 + kp) = 0$

Equation $k^2 - \frac{5k}{2} + \frac{25}{16} = 0$

Roots $\Rightarrow k_1 = \frac{5}{4}$

$= y(x) = e^{k_1 x} c_1 + e^{k_1 x} c_2 x$

substitute $\Rightarrow k_1 = \frac{5}{4}$

$\Rightarrow y(x) = (c_1 e^{\frac{5x}{4}} + (c_2 x e^{\frac{5x}{4}}$

Azaz Ali 11440 (8)

Q2 (2) $y''+14y'+49y= \quad y(-4)-(-1y')(-4)=5$

Sol You have entered

$$49y(x)+14\frac{d}{dx}y(x)+\frac{d^2}{dx^2}y(x)=6$$

Given the equation

$$49y(x)+14\frac{d}{dx}y(x)\cdot\frac{d^2}{dx^2}y(x)=0$$

The differential equation has the form.

$$y''+p^*y'+q^*y=0$$

where $p=14$

$q=49$

linear homogenous

$$y''+p^*y'+q^*y=0$$

First of all we should find the roots of the characteristic equation.

$$q+(k^2+kp)=0$$

$$k^2+14k+49=0$$

$$k_1=-7$$

$$y(x)=e^{k_1x}c_1+e^{k_1x}c_2x$$

subsitute

$$k_1=-7$$

The final answer $y(x)=c_1e^{-7x}+c_2xe^{-7x}$

$$c_1=\frac{9}{e^{28}}$$

$$c_2=-\frac{2}{e^{28}}$$

$y(x)$

$$y(x) = \left(-\frac{2x}{e^{28}} - \frac{9}{e^{28}}\right) e^{-7x}$$

The classification

nth linear constant coeff homogenous

2nd power series ordinary

The answer

$$y(x) = (c_1 + c_2 x) e^{-7x}$$

The solution of the Cauchy problem

$$y(-4) = -1$$

$$\left(\begin{cases} -4 & \text{for } 0 = 1 \\ 1 & \text{for } 1 = 1 \\ 0 & \text{otherwise} \end{cases}\right) \frac{d}{dx} y(x) \Big|_{x=-4} = 5$$

$$\frac{d}{dx} y(x) = c_2 e^{-7x} - 7(c_1 + c_2 x) e^{-7x}$$

$$y(x) = (c_1 + c_2 x) e^{-7x}$$

$$5 = c_2 e^{--28} - 7(c_1 + (-4) c_2) e^{--28}$$

$$-1 = (c_1 + (-4)(c_2) e^{--28}$$

$$c_1 = -\frac{9}{e^{28}}$$

$$c_2 = \frac{-2}{e^{-28}}$$

$$y(x) = \left(\frac{-2x}{e^{20}} - \frac{9}{e^{28}}\right) e^{-7x}$$

$$\Rightarrow y(x) = (c_1 + c_2 x) e^{-7x}$$

Q2 3 $y'' - \cancel{\#\#}4y' + 9y = 0 \quad y(0) = 0 \; y'(0) = 8$

~~You have entered~~

Solution

~~the answer~~ $9y(x) - 4\frac{d}{dx}y(x) + \frac{d^2}{dx^2}y(x) = 0$

~~y(x) = C1 sin~~

Solution $\cancel{y'' + p * y + q * y} = 0.$

~~9y(x)~~ where $p = -4$

$q = 9$

To find roots $q(k^2 + 1 + p) = 0$

equation $k^2 - 4k + 9 = 0$

Roots $k_1 = 2 - \sqrt{5}i$

$k_2 = 2 + \sqrt{5}i$

$= y(x) = e^{k_1 x}C_1 + e^{k_2 x}C_2$

$= y(x) = C_1 e^{x(2-\sqrt{5}i)} + C_2 e^{x(2+\sqrt{5}i)}$

The Solution Cauchy Problem

$y(0) = 0$

$\left( \begin{matrix} 0 & \text{for} & 0 = 1 \\ 1 & \text{for} & 1 = 1 \\ 0 & \text{otherwise} & \end{matrix} \right) \quad \frac{d}{dx} y(x) \Big|_{x=0} = -8$

$\frac{d}{dx} y(x) = 2(C_1 \sin\sqrt{5}x) + C_2 \cos(\sqrt{5}x))e^{2x}$

~~y(x)~~ =

$$y(x) = (c_1 \sin \sqrt{5}x) + c_2 \cos(\sqrt{5}x))e^{2x}$$

$$-8 = 2(c_1 \sin(0\sqrt{5}) + c_2 \cos(0\sqrt{5}))e^{0\cdot 2} + \sqrt{5}$$

$$0 = (c_1 \sin(0\sqrt{5}) + c_2 \cos(0\sqrt{5}))e^{0\cdot 2}$$

$$C_2 = 0$$

$$C_1 = \frac{-8\sqrt{5}}{5}$$

$$y(x) = \frac{-8\sqrt{5}e^{2x} \sin(\sqrt{5}x)}{5}$$

Q2 (4) $y''-8y+17y=0 \quad y(0)=-4y'(0)=-1$

sol $17y(x)-8\frac{d}{dx}y(x)+\frac{d^2}{dx^2}y(x)=0$

This differential equation has the form

$y''+p*y+q*y=0$

where $p=-8$

$q=17$

$y''+p*y+q*y=0$

$q+(k^2+kp)=0$

$k^2-8k+17=0$

$k_1=4-i$

$k_2=4+i$

$y(x)=e^{k_1x}C_1+e^{k_2x}C_2$

the final answer

$y(x)=C_1e^{x(4-i)}+C_2e^{x(4+i)}$

$y(x)(C_1\sin(x)+C_2\cos(x))e^{4x}$

Q3 Define Laplace transform along with examples

Ans In mathematics the Laplace transform, named after its inventor Pierre-Simon laplace. is a integral transform that converts a function of a real variable to a function of a complex variable. the transform has many applications in science and engineering because it is a tool for solving differential equations.

for example $f(t) = \cos(\omega_0 t)$

$$F(s) = s/(s^2+\omega_0^2)$$

(A)(1) $f(t) = 6(e^{-5t}) + e^{3t} + 5(t^3) - 9$

sol

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

Q3 2

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t$

sol $G(s)$ $4 \frac{s}{s^2+(4)^2} - 9 \frac{4}{s^2+(4)^2} + 2 \frac{8}{s^2+(10}$

$\frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$

Q3 3 $h(t) = e^{3t} + \cos(6t) - e^{(3t)}\cos($

sol $\cancel{h(t)}$ $H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6}$

$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$

Azaz Ali 11440 (15)

Q4 (1) Solve the following IVP using Laplace Transform.

(1) $y'' - 10y' + 9y = 5t, \quad y(0) = -1, \quad y'(0) = 2$

Sol Applying the Laplace transform to both side. we find

$(s^2 - 10s + 9)Y + s = 2 - 10 - \frac{5}{s^2} \Rightarrow Y(s) = \frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)}$

$$\frac{5 + 12s^2 - s^3}{s^2(s-9)(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

we find $B = \frac{5}{9}, \; D = -2, \; C = \frac{31}{81}, \; A = \frac{50}{81}$

therefore using the linearity of the inverse Laplace transform.

$$y(t) = \frac{50}{81} + \frac{5t}{9} + \frac{31}{81}e^{9t} - 2e^{t}.$$

Q4 (2) $y'' - 6y' + 15y = 2\sin(3t),\ y(0) = -1,\ y'(0) = -$

we have

$$(s^2 - 6s + 15)Y + s - 2 = \frac{6}{s^2+9} \Rightarrow Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)} = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s}$$

$$s^3: A + C = -1$$
$$s^2: -6A + B + D = 2$$
$$s^1: 15A - 6B + 9C = -9$$
$$s^0: 15B + 9D = 24$$

The solution is $A = \frac{1}{10},\ B = \frac{1}{10},\ C = -\frac{11}{10},\ D = \frac{5}{2}$

Hence we get

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

Now we need to find the inverse Laplace transform

Let us start with the first term.

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+9} + \frac{1}{s^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$= \cos 3t + \frac{1}{3}\sin 3t.$$

The second term is slightly more involved.

$$\frac{-11s+25}{s^2-6s+15} = \frac{-11s+25}{(s-3)^2+6}$$
$$= \frac{-11(s-3)-8}{(s-3)^2+6}$$
$$= -11\frac{(s-3)}{(s-3)^2+6} - \frac{8}{\sqrt{6}}\frac{\sqrt{6}}{(s-3)^2+6}$$

Now $\mathcal{L}^{-1}\left\{\frac{-11s+25}{s^2-6s+15}\right\} = -11e^{3t}\cos\sqrt{6}t - \frac{8}{\sqrt{6}}e^{3t}\sin\sqrt{6}t$

The final answer hence is

$$y(t) = \mathcal{L}^{-1}\{Y\} = \frac{1}{10}\left(\cos 3t + \frac{1}{3}\sin 3t - 11e^{3t}\cos\sqrt{6}t - \frac{8}{\sqrt{6}}e^{3t}\sin\right.$$