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Page-1 linear Algebra

Question = 1

Consider the following vector R^3

$$v_1 \begin{pmatrix} ID1 \\ ID2 \\ ID3 \end{pmatrix}, \quad v_2 \begin{pmatrix} ID2 \\ ID3 \\ ID4 \end{pmatrix}, \quad v_3 = \begin{pmatrix} ID3 \\ ID4 \\ ID5 \end{pmatrix}$$

$S = \{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$

$\vec{v}^2 = (2\vec{v}_2 + (3\vec{v}^3$

$v^2 = (1v^1 + (3v_3$

the set "S" is linearly dependent because this relation $ID1 \cdot \vec{v}_1 \quad ID3 - \vec{v}_2 - ID5\,\vec{v}_3 = 0$

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Question=2

Ans:

We constant a write Cost matrix "U" having two Products X and y.

$$U = \begin{bmatrix} 450 & 400 \\ 250 & 350 \\ 150 & 150 \end{bmatrix} \begin{matrix} \text{material} \\ \text{labor} \\ \text{overhead} \end{matrix}$$

let $x = \begin{bmatrix} x_1, ~~x_2~~ \\ x_2 \end{bmatrix}$ be a" production vector Corresponding to x_1 many of produced ~~to~~ B and x_2 ~~&~~ many of produced C. and define $T: R^2 \longrightarrow R^3$ by

$$T(x) = Ux = x_1 \begin{bmatrix} 450 \\ 250 \\ 150 \end{bmatrix} + x_2 \begin{bmatrix} 400 \\ 350 \\ 150 \end{bmatrix} = \begin{bmatrix} \text{total material cost} \\ \text{total labor cost} \\ \text{total cost overheads} \end{bmatrix}$$

Part B= if Production is increased by
(a) factor of. say. 4 from x to 4x, then the Costs will increase by the Same factor. from $T(x)$ to $4T(x)$

(b) If x and y are Production vectors, then the total cost vector associated with the Combined Production x+y is precisely the Sum of the cost $T(x)$ $T(y)$

Question-3:

four main thing vector Space.
Four main ~~types~~ thing of vector Space.

1, the Set V is Closed under vector addition, that is, $\vec{v}+\vec{w}$, $\vec{v} \in V$

2, vector addition is Commutative, $\vec{v}+\vec{w} = \vec{w}+\vec{v}$

3, vector addition is associative.
$(\vec{v}+\vec{w})+\vec{u} = \vec{v}+(\vec{w}+\vec{u})$

4, there is a zero vector $\vec{0} \in V$ Such that $\vec{v}+\vec{0}=\vec{v}$ for all $\vec{v} \in V$.

Part a: ~~V~~
$V = \{2 \times 2$ matrices with entries in $R\}$; usual matrix addition and
$k \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix}$ for $k \in R$

This is not vector Space
over R.
To make it vector Space.

$k \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

We write $M_{n \times m}$ for vector Space of $n \times m$ matrices under the natural operation and ~~matrices~~ Scaler multiplication.

Part b:

it is a vector space under the operations

$P_3 = (a_0 + a_1 x + a_2 x^2 + a_3 x^3)$

$r \cdot P_3 = r \cdot (a_0 + a_1 x + a_2 x^2 + a_3 x^3)$

$= (r a_0) + (r a_3) x + (r a_2) x^2 + (r a_3) x^3$

if we identify these two spaces elements in this way

$a_0 + a_1 x + a_2 x^2 + a_3 x^3$

Corresponds $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$

Question- 4

Determinants : let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is 2x2 matrix

(a). For which value of det M does M have an inverse

Ans=

Solution-

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= ad - cb$$

if $\det M = ad \div cb \neq 0$, when M have an inverse

(b) write down all 2x2 bit matrices with determinant 1 (Remember bit are either 0 or 1 $1+1=0$ in bits

Ans=

Sol=

$$A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Proof:

$$\det A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 1 \times 1 - 0 \times 0$$

$\det A = 1$

proved

$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Proof:

det $B = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$= 1 \times 1 = 1 \times 0$

det $B = 1$

Proved

$C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Proof:

det $C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= 1 \times 1 - 0 \times 1$

det $C = 1$

Proved

(C) write down 2x2 bit matrices with determinent

Ans:

Sol: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Proof

det $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

det $= 0 \times 0 - 0 \times 0$

det $A = 0$

Proved

$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Proof:

$\det B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= 0 \times 0 - 0 \times 1$

$\det B = 0$

proved

$C = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

Proof:

$\det C = 0 \times 1 - 0 \times 1$

$\det C = 0$

Proved

$D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Proof

$\det D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$= 0 \times 0 - 1 \times 0$

$\det D = 0$

proved

$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

Proof:

$\det E = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}$

$= 1 \times 0 - 1 \times 0$

$\det E = 0$

proved

$F = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$

det $F = 1\times 0 - 0\times 0$

$= 0$

Proved

$G = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$

Proof:- det $G = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$

$= 0\times 1 - 0\times 0$

det $G = 0$

Proved

Solution:

det $A = \begin{vmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{vmatrix}$

Expand by Row1

$ID1 \begin{vmatrix} ID3 & ID2 \\ ID1 & ID5 \end{vmatrix} - ID1 \begin{vmatrix} ID2 & ID2 \\ ID4 & ID5 \end{vmatrix}$

$+ ID1 \begin{vmatrix} ID2 & ID3 \\ ID4 & ID1 \end{vmatrix}$

$= ID1(ID3ID5 - ID2ID1) - ID1(ID2ID5 -$
$ID2ID4 + (ID1(ID2ID1 - ID3ID4)$

$= ID1ID3ID5 - ID1ID1ID2 - ID1ID2ID5$
$+ ID1ID2ID4 + ID1ID1ID2$
$- ID1ID3ID4$