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Section A

Semester 6^{th}

Paper Hydraulic Engr.

Q1

Sol:- The pressure drop (ΔP) is expected to depend upon gate opening h, and overall depth d, the velocity (V), density (ρ) and viscosity μ.

Revelant variables are $\Delta P, h, d, V, \rho, \mu$

Dimensions are given

ΔP	$ML^{-1}T^{-2}$
h	L
d	L
V	LT^{-1}
ρ	ML^{-3}
μ	$ML^{-1}T^{-1}$

We have

Number of variables $n=6$

Number of independent dimension $m=3$ (M, L and T)

Number of non-dimensional groups $n-m=3$.

Choose $m(=3)$ scaling variables

geometric (d), kinematic/time dependent (V), dynamic/mass dependent.

From dimensionless group by non-dimensionalising remaining variables ΔP, h, μ.

$$\pi_1 = \Delta p\, d^a V^b \rho^c$$

$$M^0L^0T^0 = (ML^{-1}T^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c$$

$$= M^{1+c}\, L^{-1+a+b-3c}\, T^{-2-b}$$

$M \Rightarrow 0 = 1+C \qquad C=-1$

$T \Rightarrow 0 = -2-b \qquad b=-2$

$L \Rightarrow 0 = -1+a+b-3c \qquad a = 1+3c-b = 0$

$$\pi_1 = \Delta p V^{-2} \rho^{-1} = \frac{\Delta P}{\rho V^2}$$

$\pi_2 = h/d$ (by inspection, since h is a length).

$$\pi_3 = \mu d^a V^b \rho^c$$

$$M^0L^0T^0 = (ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-1-b}$$

$M \Rightarrow \quad 0 = 1+c \quad \Rightarrow c = -1$

$T \Rightarrow \quad 0 = -1-b+0 \Rightarrow b = -1$

$L \Rightarrow \quad 0 = -1+a+b-3c \Rightarrow a = 1+3c-b = -1$

$$\pi_3 = \mu d^{-1} V^{-1} \rho^{-1} = \frac{\mu}{\rho V d}$$

Recognition of Reynold's number suggest that we replace π_3 by

$$\pi_3' = (\pi_3)^{-1} = \frac{\rho V d}{\mu}$$

By dimensional analysis.

$$\pi_1 = f(\pi_2, \pi_3')$$

$$\frac{\Delta P}{\rho V^2} = f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

a) Dynamic similarity requires that all non-dimensional group be same in model and prototype.

$$\pi_1 = \left(\frac{\Delta P}{\rho V^2}\right) = \left(\frac{\Delta P}{\rho V^2}\right)$$

$\pi_2 = (h/d) = (h/d)$ (automatic if similar shape " geometric similarity")

$$\pi_3' = \left(\frac{\rho V d}{\mu}\right) = \left(\frac{\rho V d}{\mu}\right)$$

We have a velocity ratio

$$\frac{V_p}{V_m} = \frac{(u/p)_p}{(u/p)_m}\frac{d_m}{d_p} = \frac{0.002/800}{1.0\times10^{-6}} \times \frac{1}{5} = 0.5$$

Hence

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5} = 6.0 ms^{-1}$$

b) Ratio of quantities of flow

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{area})_p}{(\text{velocity} \times \text{area})_m}$$

$$= \frac{V_p}{V_m}\left(\frac{d_p}{d_m}\right)^2$$

$$= 0.5 \times 5^2$$

$$= 12.5$$

c) Pressure drop

$$\pi_1 = \left(\frac{\Delta P}{\rho V^2}\right) = \left(\frac{\Delta P}{\rho V^2}\right)$$

$$\frac{(\Delta P)_p}{(\Delta p)_m} = \frac{P_p}{P_m}\left(\frac{V_p}{V_m}\right)^2$$

$$= \frac{800}{1000} \times 0.5^2$$

$$= 0.2$$

Now

$$\Delta P_p = 0.2 \times \Delta p_m$$

$$= 0.2 \times 60$$

$$= 12.0 kPa.$$

Q2

Sol:-

Maximum depth of water in Reservior = H_w = 78m
Specific Gravity of dam material = G = 2.6
Allowable compressive stress for Dam Masonary = 781 T/m²
Height of wave = 4.2m.
No uplift pressure = Cu = 0
μ = 0.7

1. H limiting $= \dfrac{\sigma_{au}}{\gamma_W (G - Cu + 1)}$

Put values

$$H_{limiting} = \frac{781 \times 1000}{1000(2.6 - 0 + 1)} = 216.9\,m > H_W = 78m.$$

So it is low gravity dam.

2. Top width "a"

Free board = 1.5 h wave = 1.5 × 4.2
= 6.3m.

Height of dam = $H_D = H_W + F.B$
= 78 + 6.3
H_D = 84.3 m.

a = 14% of H_D

a = 0.14 × 84.3 = 11.8 m.

3. Base Width "b" (Without offset)

i- For No Sliding Criteria

$$b' = \frac{H_w}{\mu G} = \frac{78}{0.7 \times 2.6} = 42.85 \simeq 43m$$

ii) For No tension criteria.

$$b' = \frac{H_W}{\sqrt{G}} = \frac{78}{\sqrt{2.6}} = 49m.$$

4) Depth of vertical portion on U/s side.

$$h' = 2a\sqrt{G - C_u}$$

$$= 2 \times 11.8 \sqrt{2.6 - 0}$$

$$h' = 38m$$

5) Upstream offset

$$a/16 = \frac{11.8}{16} = 0.73m.$$

6) Depth below water level to end of inclined portion in US $= 3.14\, a\sqrt{G}$

$$= 3.14 \times 11.8 \sqrt{2.6}$$

$$= 59.7m.$$

7) Total width of base of dam

$$b = b' + a/16$$

$$= 49 + 11.8/16 = 49.7m.$$

8) $\tan\theta = b'/H = \frac{49}{78} = \tan^{-1}(0.62)$

$$= 32.13^\circ$$

9) Depth of vertical portion on D/s (from WL on U/s side.

$$\tan\theta = a/d' \quad = \frac{11.8}{d'}$$

$$\frac{49}{78} d' = 11.8$$

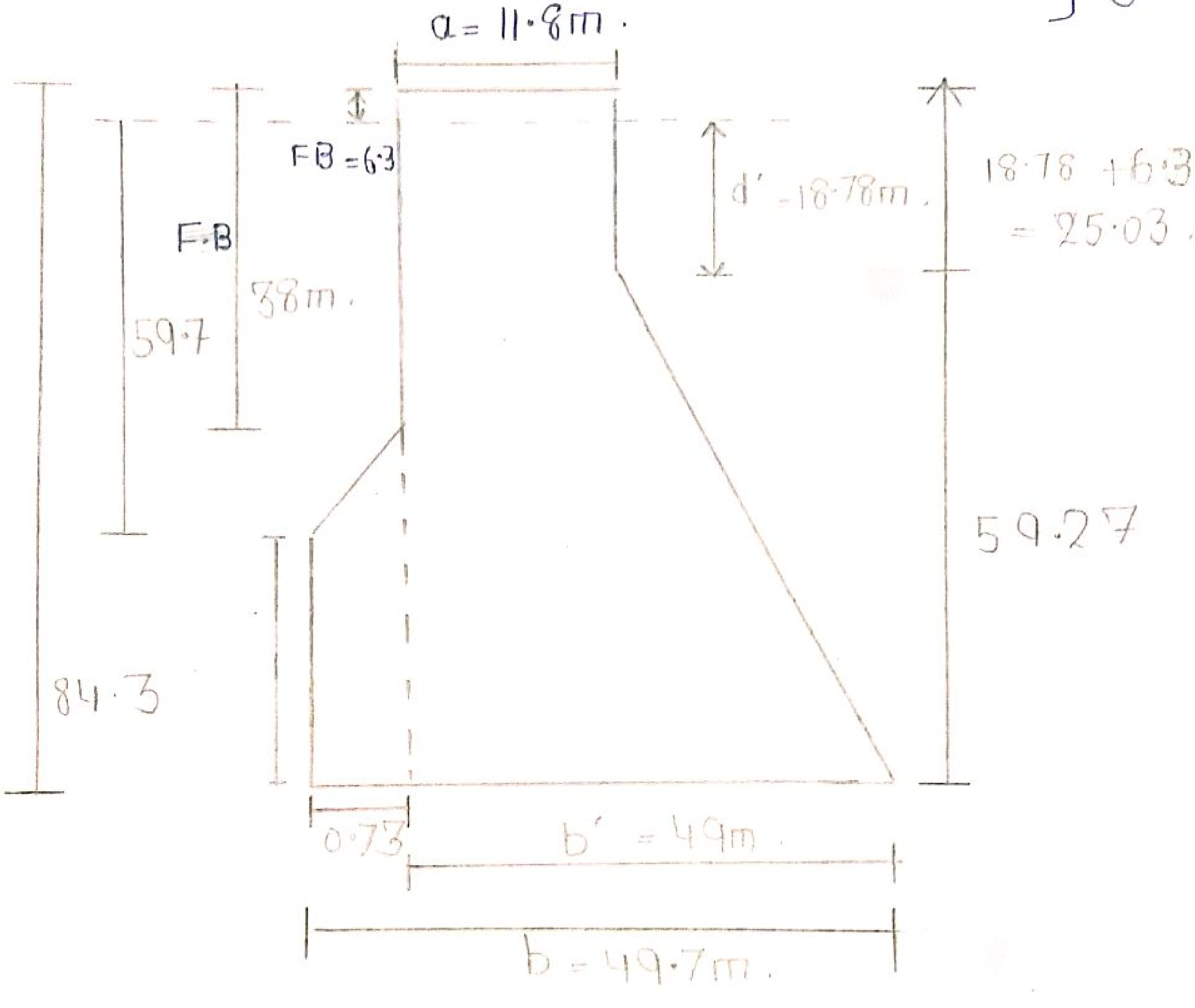
$$d' = \frac{11.8 \times 78}{49}$$

$$= 18.78m.$$

Depth of vertical portion

$$d = d' + F_B$$

$$= 18.78 + 6.3 = 25.03m.$$



a = 11.8m.
FB = 6.3
F.B
38m.
59.7
84.3
d' = 18.78m.
18.78 + 6.3
= 25.03.
59.27
0.73
b' = 49m.
b = 49.7m.

Q3.

Dimension Analysis:-

If certain physical phenomena is governed by
$f(x_1, x_2, \ldots x_n) = 0$
Where some/all of variables are dimensional
Then above phenomena can be represented as.
$(\pi_1, \pi_2, \ldots \pi_m) = 0$ Where all variables (π) are non-dimensional.

Buckingham Pi Theorem:-

$f(x_1, x_2, \ldots x_n) = 0 \implies \psi(\pi_1, \pi_2, \ldots \pi_m) = 0$

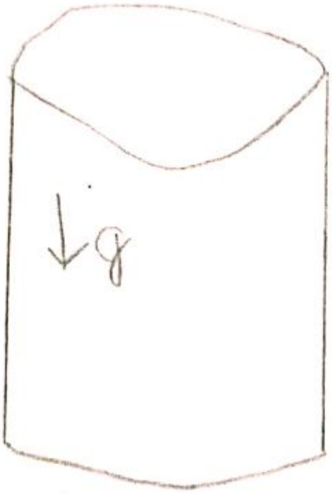
Where some/all x are dimensional; where all π are non-dimensional.

Where $m < n$, $m = n - k$.

Experiment shows for viscous flow

$$f(v, g, h, \nu) = 0$$

	M	L	T
v	0	1	-1
g	0	1	-2
h	0	1	0
ν	0	2	-1



$n = 4 \quad k = 2, \; m = 2$

	L	T
V		
g	1	-2
h	1	0
ν	2	-1

$f(V, g, h, \nu)$ $n=4, k=2, m=2$

Repeating variable - g, h

Non-repeating variable V, ν

$$\pi_1 = x_m (x_1)^{a_1} (x_2)^{a_{12}} (x_3)^{a_{13}} \cdots (x_{rk})^{a_{13}}$$

$$\pi_1 = V (g)^a (h)^b$$

$$= L^0 T^0 = LT^{-1} (LT^{-2})^a (L)^b = L^{1+a+b} T^{-1-2a}.$$

$$a = b = -1/2 \quad \pi = V/\sqrt{gh}.$$

Similarly

$$\pi_2 = \nu (g)^a (h)^b$$

$$L^0 T^0 = L^2 T^{-1} (L)^b.$$

$$2 + a + b = 0 = -1 - 2a$$

$$a = -1/2, \ b = -3/2$$

$$\pi_2 = \nu/\sqrt{gh}.$$

$$f(V, g, h, \nu) = 0 \quad f_1\left(\frac{V}{\sqrt{gh}}, \frac{\nu}{\sqrt{gh^3}}\right) = 0$$

$\frac{V}{\sqrt{2gh}} = Fr$ Froud number

$$\frac{\nu}{\sqrt{gh^3}} = \frac{V}{\sqrt{gh}} \frac{\nu}{Vh} = \frac{Fr}{Re}.$$

We may also write

$$f_2(Fr, Fr/Re) = 0 \quad Fr = \psi(Fr/Re).$$

Viscous flow

$$Fr = \psi(Fr/Re)$$

Advantages of Dimension Analysis

$$f(V, g, h, \nu) = 0 \Rightarrow Fr = \psi\,(Fr/Re)$$

$$Fr = V/\sqrt{gh} \qquad \frac{Fr}{Re} = \frac{\nu}{\sqrt{gh^3}}.$$

- Less number of experiments are necessary
- Experiment becomes inexpensive.
- Data reduction becomes easier.

Dimension analysis show

$$Fr = \psi(Fr/Re).$$

Q 4:

Fall velocity:-

When a grain falls down in still water it obtains a constant velocity when forward fluid drag force on the grain is equal to downward submerged weight of grain.
This constant velocity is defined as grain fall velocity of grain.
This is also called setting velocity.

Particle Density:-

Particle density effect fall velocity. As with decreasing altitude air density increases at rate of 1%. per 80 meters.

Viscosity of Water:-

Fluid velocity through porous media is approximately as inversely to kinematic viscosity. A decrease in viscosity therefore causes increase in velocity.

Turbulance of water:-

Turbulance effect fall velocity of water in reservior because non-linearity and zig zag path effect flow of water and causes variation.

Particle Shape:-

Non-spherical particle fall upto 75% slower than equivalent sphere model Non-spherical particle travels 44% faster spherical particle.

Particle concentration:-

When suspended concentration of sediment increases the velocity of each particle decreases due to modification of flow induced by other particles.

Particle diameter:- The diameter of a sphere particle have same specific gravity and terminal uniform setting velocity as given particle in same sedimentation.