Iqra National University Peshawar

Name : Waqar Ul Mulk

ID : 7727

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Submitted to: Miss Shomaila Mazhar

Q No 1

Solve the Intial Value Problem

$dy/dt = e^y \_t \sec(y)(1+t^2)$

$y = (0) = 0$

Solution;

$dy/dt = e^{y-t} \sec(y)(1+t^2)$

$y(0) = 0$ So, $x = 0$
$y = 0$

$dy = e.e \sec(y)(1+t^2) dt$

$\frac{1}{e^y . \sec(y)} = (1+t^2) e^{-t} dt$

As $\cos(y) = \frac{1}{\sec(y)}$

$\int e^{-y} \cos y \, dy = \int (1+t^2) e^{-t} dt$

Using Integration by Parts

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$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \frac{d}{dy} e^{-y} \right) =$$

$$(1+t^2) \int e^{-t} - \int \left(\int e^{-t} \cdot \frac{d}{dt} (1+t^2) \right)$$

↳ eq 1

L.H.S

$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right)$$

$$e^{-y} \sin y - \int (\sin y . e^{-y} (-1))$$

$$e^{-y} \sin y + \int (\sin y . e^{-y})$$

$$e^{-y} \sin y + \int (\sin y . e^{-y})$$

$$e^{-y} \sin y + \int (e^{-y} \sin y)$$

Now,

Again using Integration by Parts

$$e^{-y} \sin y + e^{-y} (-\cos y) - \int \left(\int \sin y \frac{d}{dy} e^{-y} \right)$$

$$e^{-y} \sin y + e^{-y} (-\cos y) - \int \left(-\cos y \frac{e^{-y}}{-1} \right)$$

Since $\int (\cos y e^{-y}) = L.H.S$

Since it is given as same to the first one so L.H.S will become

$$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$$

$$2L.H.S = e^{-y}(\sin y - \cos y)$$

$$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$$

Now Taking R.H.S

$$\int (1+t^2)\, e^{-t}\, dt$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt}(1+t^2)\right)$$

$$-(1+t^2)\, e^{-t} - \int(-e^{-t}(2t))$$

$$-(1+t^2)\, e^{-t} + \int (2t)e^{-t})$$

Using Integration by Parts.

$$-(1+t^2)e^{-t} + (2t\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt} 2t\right))$$

$$-(1+t^2)\, e^{-t} + (2te^{-t} + \int(2e^{-t}))$$

$$-(1+t^2)\, e^{-t} + (-2te^{-t} - 2e^{-t}) + c$$

$\Rightarrow -(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c$

$\Rightarrow -e^{-t} - e^{-t} - 2e^{-t} - 2e^{-t} + c$

$\Rightarrow -(t^2+2t+3)e^{-t} + c = R.H.S$

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

we know that

$x = 0 \qquad y = 0$

$\Rightarrow \frac{1}{2}(0-1) = -3 + c$

$c = 5/2$

Put value of c.

$$\boxed{\frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2+2+3)e^{-t} + 5/2}$$

Q. No. 2.

Solve ;

$$\Rightarrow (\sqrt{x+y} + \sqrt{x-y})\,dx - (\sqrt{x+y} - \sqrt{x-y})\,dy = 0$$

$$\Rightarrow dy/dx = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \quad \text{—— (1)}$$

This is homogenous Differential eq in x & y to solve this put

$$\Rightarrow dy/dx = v + x\, dv/dx$$

Thus eq (1) becomes

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1+\sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

Taking Integral on b/s

$$\Rightarrow \int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2}=t$

$\Rightarrow \Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$

$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}} = -dt$

$\Rightarrow \int \frac{-dt}{t} = \int \frac{dx}{x}$

$\Rightarrow -\ln t = \ln x + \ln c$

$\Rightarrow -\ln(1+\sqrt{1-v^2}) = \ln cx$

$\Rightarrow \ln(1+\sqrt{1-v^2} = -\ln cx$

$\Rightarrow \cancel{\ln}(1+\sqrt{1-v^2} = \cancel{\ln}((cx)^{-1}$

$\Rightarrow 1+\sqrt{1-v^2} = \frac{1}{cx}$

$\Rightarrow 1+\sqrt{1-y^2/x^2} \quad \frac{1}{cx}$

$\Rightarrow 1+\sqrt{\frac{x^2-y^2}{x^2}} = 1/cx$

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$\Rightarrow x = \sqrt{x^2 - y^2} = \frac{1}{c}$

$\Rightarrow x + \sqrt{x^2 - y^2} = c_1 \quad \because \frac{1}{c} = c_1$

which is required solution

Q. No. 3

Solve,

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

Solution,

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow f(D)y = f(x)$$

As It is non homogenous linear equation so solution will be

$$y = y_c + y_p \quad \text{— (i)}$$

Complementary solution y_c

$$D^4 + D^2 = 0 \quad \Rightarrow D^2(D^2+1) = 0$$

either

$$D^2 = 0 \quad \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \quad \Rightarrow D = i \text{ or } D = 0 - i$$

Root are real & complex

$y_c = C_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x$

$y_p = \frac{1}{f(D)} f(x$

$y_p = \frac{1}{D^4 + D^2}(3x + 4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 + D^2}$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

So, $f'(D) = 4D^3 + 2D$

Now also for $D = 0 = f'(D) = 0$

Again Differentiating ;

$f''(D) = 12D + 2$

So for $D = 0$

$f''(D) = 12(0) + 2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow yp = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} 4\sin x \rightarrow$$

$$\rightarrow -\frac{x^2}{12D+2} 2\cos x$$

Putting $D = 0$ in all

So,

$$yp = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 . 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$yp = \frac{3x^4}{2} + \frac{4x\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So, Putting in eq (1)

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$y = C_1 + C_2 \cos x + C_3 \cos x + 3/2\, x^4 + 2x^2 \sin x$

$$\boxed{y = C_1 + (C_2 - x^4)\cos x + (C_3 + 2x^2)\sin x + \tfrac{3}{2}x^4}$$

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