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(1)

Q 1 (a)

~~Ans~~ As we have:

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} \, dt$$

Differenting B/S:

$$\frac{dX}{dw}(jw) = \int_{-\infty}^{\infty} jt \, x(t) e^{-jwt} \, dt$$

$$\frac{dx}{dw}(jw) = -jt \int_{-\infty}^{\infty} x(t) e^{-jwt} \, dt$$

As: jt is constant

$$\frac{dX}{dw}(jw) = -jt \, F(x(t))$$

$$-jt \, x(t) \xleftrightarrow{F} \frac{d}{dt} X(jw)$$

Q# 1 (b)

Given:

$x[n] = 2\delta[n] - 4\delta[n-2] +$
$2\delta[n-3]$.

$h[n] = 3\delta[n] + \delta[n-1] +$
$2\delta[n-2]$.

find

$Y[z]$ and $Y[n]$.

Sol:

$X[z] = 2 - 4z^{-2} + 2z^{-3}$

$h[z] = 3 + z^{-1} + 2z^{-2}$

Now for $Y[z]$ we have multiply $H[z]$ to $X[z]$.

So:

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$Y[z] = H[z] * X[z]$

$$Y[z] = (2 - 4z^{-2} + 2z^{-3})(3 + 1z^{-1} + 2z^{-2}]$$

$$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 3z^{-4}$$

$$+ 6z^{-3} + 2z^{-4} + 4z^{-5}$$

$$= 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$$

For $y[n]$ we have:

delay property:

$$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2]$$

$$+ 2\delta[n-3] - 6\delta[n-4] +$$

$$4\delta[n-5]$$

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Q#2 Ans:

$$f(x) = \begin{cases} -\pi/2 & -\pi \leq x < 0 \\ \pi/2 & 0 \leq x \leq \pi \end{cases}$$

As we now for

fourier series we have

to a_n and b_n so

first we have a_n:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

Now limit is '$-\pi$ to 0'
'0 to π'

and value of $f(x) = -\pi/2$

so:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{0} -\frac{\pi}{2}\,dx + \int_{0}^{\pi} \frac{\pi}{2}\,dx$$

By apply integral:

$$\frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} 1 \cdot dx + \frac{\pi}{2}\int_{0}^{\pi} 1\,dx\right.$$

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$$\frac{1}{\pi}\left[-\frac{\pi}{2}(x)\Big|_{-\pi}^{0}+\frac{\pi}{2}(x)\Big|_{0}^{\pi}\right.$$

$$=\frac{1}{\pi}\left[-\frac{\pi}{2}(0)-(-\pi)\right]+\left[\frac{\pi}{2}(\pi)-(0)\right].$$

By appling limit:

So:

$$=\frac{1}{\pi}\left[-\frac{\pi^2}{2}+\frac{\pi^2}{2}\right].$$

$$=\frac{1}{\pi}[0]$$

$$a_n = 0$$

Now bn:

For bn we have:

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2} \sin nx\,dx + \int_{0}^{\pi} \sin nx\,dx\right]$$

As $\int \sin nx = -\cos nx$

So:

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}(-\cos nx)\Big|_{-\pi}^{0} + \frac{\pi}{2}(-\cos nx)\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi n}\left(-\frac{\pi}{2}(-\cos n(0) - (-\cos n(-\pi))\right.$$

$$\left. + \frac{\pi}{2}(-\cos(\pi) - (-\cos(0))\right]$$

By appling limits:

$$= \frac{1}{\pi n}\left[-\frac{\pi}{2}(-2) + \frac{2\pi}{2}\right]$$

$$= \frac{1}{n\pi}\left[\frac{2\pi}{2} + \frac{2\pi}{2}\right]$$

$$= \frac{1}{n\pi}\left[\frac{4\pi}{2}\right]$$

$$= \frac{4\pi}{n2\pi} \quad \frac{1}{2n}$$

$$= \frac{1}{2n}$$

$$\Rightarrow \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{1}{2n} & \text{if } n \text{ is odd} \end{cases}$$

So :

$$= 0 + 0 + 0 + \text{---} \quad \frac{1}{2} \sin x$$

$$+ \frac{1}{4} \sin 2x + \frac{1}{6} \sin 3x + \frac{1}{8} \sin 4x \text{ ---}$$

Now coefficient of

a_n .

$$a_n = \frac{1}{\pi} \int_{-\pi}^{0} -\frac{\pi}{2} \cos nx \, dx + \int_{0}^{\pi}$$

$\cos nx \, dx$

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As $\int \cos nx = \sin nx$

So:

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2} (\sin nx) \Big|_{-\pi}^{0} + \frac{\pi}{2} (\sin nx) \Big|_{0}^{\pi} \right]$$

By Applying intregation:

Now appling limits:

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2} (\sin(0) - \sin n(-\pi)) + \frac{\pi}{2} (\sin n(\pi) - \sin n(0)) \right].$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0) \right].$$

$$= \frac{1}{n\pi}\left[0 + 0 \right].$$

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$$= \frac{1}{n\pi} [0].$$

$$= 0$$

So b_n is become:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

So we take

$b_n = 0$ and

$a_n = 0.$

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(A)

Q #3 Ans

Given:

$$x[z] = \frac{2z^2+2z}{(z^2+2z-3)}$$

Sol:

$$= \frac{2z^2+2z}{(z^2+2z-3)}$$

$$= \frac{2z^2+2z}{(z-1)(z+3)}$$

By factors

$$= \frac{2z^2+2z}{(z-1)(z+3)} = \frac{A}{(z-1)} + \frac{B}{(z+3)} \rightarrow (A)$$

xing $(z-1)(z+3)$

B/S

$$\frac{2z^2+2z}{(z-1)(z+3)} \times (z-1)(z+3) =$$

$$\frac{A}{z-1} \times (z-1)(z+3) + \frac{B}{z+3} \times (z-1)(z+3)$$

$\Rightarrow 2z^2+2z = A(z+3) + B(z-1) \rightarrow (i)$

Now we have 2

values

$z = -3, \quad z = 1$

put $z = -3$ in equ (i)

$2z^2+2z = A(z+3)+B(z-1)$

$2(-3)^2+2(-3) = A(-3+3)+B(-3-1)$

$\Rightarrow 2(9)-6 = A(0)+B(-4)$

$18-6 = 0+B(-4)$

$12 = -4B$

$\div$ing -4 B/S

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$$\frac{\cancel{12}^{3}}{\cancel{-4}_{1}} = \frac{-\cancel{4}B}{\cancel{-4}B}$$

$$\boxed{B = -3}$$

put $z = 1$ in equ (i).

$$2(1)^2 + 2(1) = A(1+3) + B(1-1)$$

$$2+2 = A(4) + B(0).$$

$$4 = A4$$

÷ing 4 on B/S

$$4/4 = \frac{A4}{4}$$

$$\boxed{A = 1}$$

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putting 'A' and 'B' values in (A).

Now:

$$\frac{2z^2+2}{(z-1)(z+3)} = \frac{1}{(z-1)} - \frac{(-3)}{(z+3)}.$$

Now:

$$X[z] = 1\frac{z}{(z-1)} - (-3)\frac{z}{(z+3)}.$$

B. For 'Z' transform:

Now inverse 'Z' transform is:

$$\boxed{x[n] = 1\, u[n] - 3(-3)^k}$$

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Q # 4 Ans

Given:

$$A \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

As we know

$$G[S] = C(SI - A)^{-1} B + D.$$

putting values:

$$G[S] = [1 \ 2]\left[S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0].$$

$$= [1 \ 2]\left[\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= [1 \ 2] \begin{bmatrix} S+2 & 1 \\ 0-1 & S-0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0].$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & 2s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$\boxed{G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}}$$

Q # 5 Ans:

$$x(t) = e^{-a|t|} \qquad a>0$$

The fourier transform of given function is:

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$$

$$X(jw) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-jwt} dt$$

$\because x(t) = e^{-a|t|}$

So:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} & \text{for } t < 0 \end{cases}$$

by figure:

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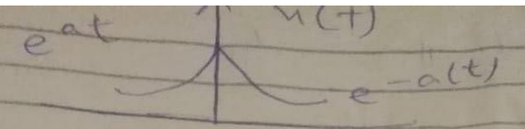

$$X(jw) = \int_{-\infty}^{0} e^{at} e^{-jwt}\, dt + \int_{0}^{\infty} e^{-at} e^{-jwt}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-jwt}\, dt + \int_{0}^{\infty} e^{-at} e^{-jwt}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-jwt)}\, dt + \int_{0}^{\infty} e^{-(a+jw)t}\, dt$$

base same power add:

By appling intregation:

$$= \frac{e^{(a-jw)t}}{a-jw}\Big|_{-\infty}^{0} + \frac{e^{-(a+jw)t}}{-(a+jw)}\Big|_{0}^{\infty}$$

$$\frac{1}{(a-jw)}\left(e^{0} - e^{-\infty}\right) - \frac{1}{(jw+a)}\left(e^{\infty} - e^{0}\right)$$

By appling limits.

$$\frac{1}{(a-jw)}(1)-\frac{1}{(a+jw)}(-1).$$

$$=\frac{1}{(a-jw)}+\frac{1}{(a+jw)}$$

$$=\frac{a+\cancel{jw}+a-\cancel{jw}}{a^2+(jw)^2}$$

$$\boxed{jw=\frac{2a}{a^2+(jw)^2}}$$

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