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Subject : EMF

Mid Assignment

Q 01:

(a) Transform the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Sol: $B = yi(x+z)j$

points are $(-2, 6, 3)$

$$B = yi(xj + zj)$$

$$B = yxij + yzij$$

$$\rho = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-2)^2 + (6)^2}$$

$$\rho = \sqrt{40} = \boxed{6.3}$$

$$z = z$$

So $z = 3$

As we know

$$\phi = \tan^{-1}(y/x)$$

$$\phi = \tan^{-1}(6/-2)$$

$$\phi = \tan^{-1}(-3)$$

$$\boxed{\phi = -71.56}$$

$$\boxed{B = 6.32, -71.56, 3}$$ Ans

(b) Convert points (3, 4, 5) from Cartesian to spherical coordinates.

Sol:- $P(3, 4, 5)$

$x = 3, \quad y = 4, \quad z = 5$

In spherical coordinate system

r, θ, ϕ

$r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{9+16+25}$

$r = \sqrt{50}$, $\boxed{r = 7.07}$

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(4/3)$

$\theta = \tan^{-1}(1.33)$

$\boxed{\theta = 53.1^\circ}$

$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{3^2+4^2}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$

$\phi = \tan^{-1}(5/5)$

$\phi = \tan^{-1}(1)$

$\boxed{\phi = 45}$

$r = 7.07$, $\theta = 53.1^\circ$; $\phi = 45$.

(c) Find the spherical coordinates of A(2, 3, -1)

Sol: $r = \sqrt{x^2+y^2+z^2}$

$r = \sqrt{2^2+3^2+(-1)^2}$

$r = \sqrt{14} \Rightarrow \boxed{r = 3.74}$

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(3/2)$

$\theta = \tan^{-1}(1.5)$

$\boxed{\theta = 56.3^\circ}$

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{2^2+3^2}}{-1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{4+9}}{-1}\right)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{13}}{-1}\right)$$

$$\phi = \tan^{-1}(-3.60)$$

$$\boxed{\phi = 74.4}$$

$r = 3.74$, $\theta = 56.3^\circ$, $\phi = 74.4$

(d) Find the cartesian coordinates of B (4, 25, 120)

Sol: Points B(4, 25, 120) is given in spherical (r, θ, ϕ) so we will find (x, y, z).

$$x = r \sin\theta \cdot \cos\phi$$

$$x = 4 \sin\theta^{(25)} \cdot \cos(120)$$

$$x = 4(0.42)(-0.5)$$

$$\boxed{x = -0.84}$$

$y = r \sin\theta \cdot \sin\phi$

$y = 4 \sin(25) \cdot \sin(120)$

$y = 4(0.42)(0.86)$

$\boxed{y = 1.45}$

$z = r \cos\theta$

$z = 4\cos(25)$

$z = 4(0.90)$

$\boxed{z = 3.62}$

$x = -0.84,\ y = 1.45,\ z = 3.62$

(e) Find the force between two charges when they are brought in contact and seperated by 4cm apart, charges are 2nc and -1nc in UN

Given data

$q_1 = 2nc,\ q_2 = -1nc,\ d = 4cm$

$F = ?$

Sol: $F = K \frac{q_1 q_2}{r^2}$

$K = \frac{1}{4\pi\epsilon_0}$

$F = \frac{2\times10^{-9} \times -1\times10^{-9}}{4(3.14)\times 8.85\times10^{-12} \times (4\times10^{-2})^2}$

$F = -1.124 \times 10^{-5}$

$\boxed{F = -11.24\ \mu N}$

(f) Find the electric field intensity of two charges -2C and -1C seperated by a distance 1m in air.

Given data

$q_1 = -2C$, $q_2 = -1C$, $d = 1m$

$E = ?$

Sol: $E_1 = \frac{Kq_1}{d^2}$ $K = 9\times10^9$

$E_1 = \frac{9\times10^9 \times -2}{(1)^2}$

$E_1 = -18\times10^9\ V/m$

$$E_2 = \frac{K q_2}{d^2}$$

$$E_2 = \frac{9 \times 10^9 \times (-1)}{(1)^2}$$

$$E_2 = -9 \times 10^9 \ V/m$$

$$E_T = E_1 + E_2$$

$$E_T = -18 \times 10^9 + (-9 \times 10^9)$$

$$\boxed{E_T = -27 \times 10^9 \ V/m}$$

(g) Determine the charge that produce an electric field strength of 40 V/cm at a distance of 30cm in vaccum (in $10^{-8}C$).

Given data

$E = 40 \ V/cm$, $d = 30 \ cm$

$Q = ?$

Sol:

$$E = K\frac{Q}{d^2}$$

$$\frac{Ed^2}{K} = Q$$

$$Q = \frac{Ed^2}{K}$$

$$Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$Q = \frac{40 \times (900)}{9 \times 10^9}$$

$$Q = 4 \times 10^{-6} C$$

or $\boxed{Q = 4 \mu C}$

(b) A charge of 2×10^{-7} is acted upon by a force of 0.1N. Determine the distance to the other charge of $4.5 \times 10^{-7} C$, both the charges are in vacuum.

Given data

$q_1 = 2 \times 10^{-7} C$, $q_2 = 4.5 \times 10^{-7} C$

$F = 0.1N$, $K = 9 \times 10^9$

$d = ?$

Sol:

$$F = \frac{K q_1 q_2}{d^2}$$

$$d^2 = \frac{K q_1 q_2}{F}$$

$$d^2 = \frac{9\times10^9 (2\times10^{-7})(4.5\times10^{-7})}{0.1}$$

$$d^2 = 8.1 \times 10^{-3} = 0.0081$$

Taking under root on both sides

$$\sqrt{d^2} = \sqrt{0.0081}$$

$$d = 0.09$$

$$d = 9\times10^{-2}$$

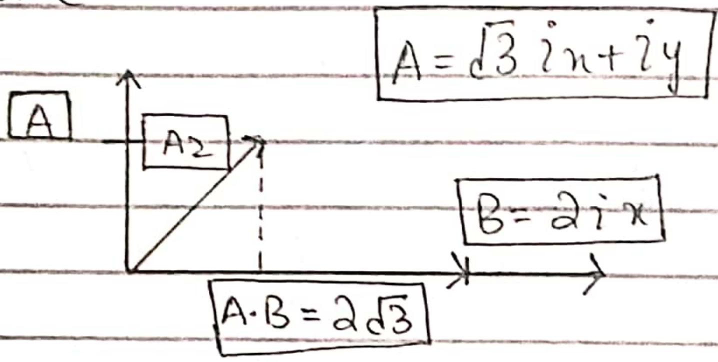
or

$$\boxed{d = 9\,cm}$$

X ——— X ——— X ——— X ——— X

Q#02 (a)

Find the angle between the vectors shown in figure.



Sol: $A = \sqrt{3}\,\hat{i}x + \hat{j}y$

$|A| = 2$

$B = 2\hat{i}x$

$|B| = 2$

$A \cdot B = 2\sqrt{3}$

$A \cdot B = |A||B| \cos\theta_{AB}$

$\cos\theta_{AB} = \frac{A \cdot B}{|A||B|}$

$\theta_{AB} = \cos^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{2} \times 2}\right)$

$\boxed{\theta_{AB} = 30^\circ}$

Q #02 (b)

Find gradient

(i) $f = ax^2 + by^3 z$

Sol: $\nabla f = \left(\frac{\partial i}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial k}{\partial z} \right)(ax^2 + by^3 z)$

$\nabla f = \frac{\partial}{\partial x} ax^2 i + \frac{\partial}{\partial y} by^3 zj + \frac{\partial}{\partial z} by^3 z$

$\nabla f = 2axi + 3bzy^2 j + by^3 k$

$\boxed{\nabla f = 2axi + 3bzy^2 j + by^3 k}$

(ii) $f = ar^2 \sin\theta + brz \cos 2\theta$

$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi} \hat{\phi}$

$\nabla f = \frac{\partial}{\partial r} (ar^2 \sin\phi + brz \cos 2\phi) \hat{r} + \frac{1}{r}$

$= \frac{\partial}{\partial \theta} (ar^2 \sin\phi + brz \cos 2\phi) \hat{\theta}$

$+ \frac{1}{\sin\theta} \cdot \frac{a}{\partial \theta} (ar^2 \sin\phi + brz \cos 2\phi) \hat{\theta}$

$$\nabla f = (2ar\sin\phi + brz\cos 2\phi) + \frac{1}{r}(0)$$

$$+ \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin\phi)\hat{\theta}$$

$$\nabla f = \frac{\partial f}{\partial p}\hat{p} + \frac{1}{p}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial A_z}{\partial z}$$

$$\nabla f = 0\hat{p} + \frac{1}{p}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}$$

$$+ (brz\cos 2\phi)\hat{z}$$

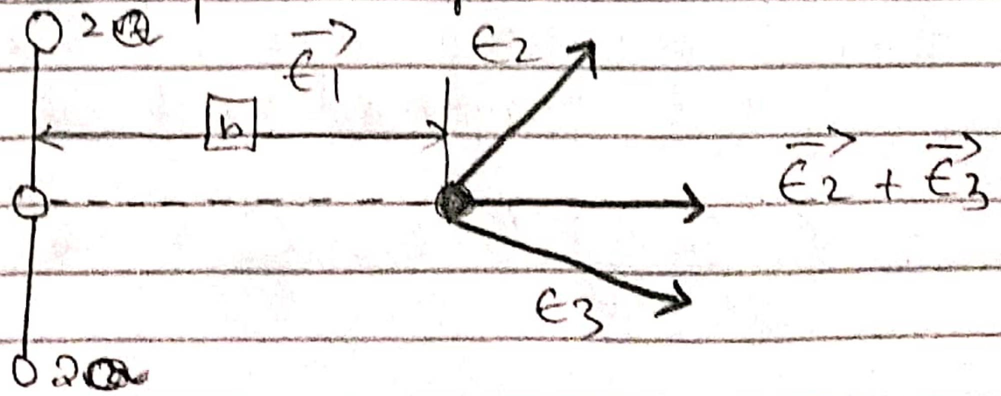
$$\nabla f = \frac{1}{p}(ar^2\cos\phi) - 2brz\sin 2\phi)\hat{\theta}$$

$$+ (brz\cos 2\phi)\hat{z}.$$

x —— x —— x —— x —— x

Q # 03

Three pointer charges are placed on the y-axis as shown. Find the electric field at point p on the x-axis.



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Sol: The distance between charge 2Q and point p is.

$$r^2 = b^2 + a^2$$

$$r = \sqrt{b^2 + a^2}$$

Assume that charge 2Q make angle (α) and ($-\alpha$) with x-axis.

Magnitude of $|\vec{E_1}| = |\vec{E_2}|$

$$= \frac{kq}{r^2}$$

$$= \frac{k(2Q)}{r^2}$$

$$= \frac{k(2Q)}{b^2 + a^2}$$

Resultant of $\vec{E_1}$ and $\vec{E_2}$ is

$$\vec{E}_{1-2} = \vec{E_1} + \vec{E_2} = \vec{E}_{1x} + \vec{E}_{2x}$$

(Y- component will be canceled)

$$= \frac{k(2Q)}{b^2 + a^2} (\cos(\alpha) + \cos(-\alpha))$$

$$= \frac{K(2Q)}{b^2+a^2}(2\cos(\alpha))$$

$$\vec{E}_{1+2} = \frac{4KQ\cos(\alpha)}{b^2+a^2} \rightarrow (i)$$

Electric field at point 'P' is due to charge "$-Q$".

As charge is negative Electric field at point will to directed toward charge "$-Q$".

$$\vec{E}_A = -\frac{K(Q)}{b^2}$$

Net electric field at point P will be

$$\vec{E}_{net} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$$

$$= \frac{-K(Q)}{b^2} + \frac{4KQ\cos\alpha}{b^2+a^2}$$

$$= \frac{-KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $k = 9 \times 10^9 \; Nm^2/C^2$

$$\vec{E}_{Net} = \frac{9 \times 10^9 \, Q}{b^2(a^2+b^2)} \left[4b^2 \cos\alpha - (a^2+b^2) \right]$$

$$\alpha = \tan^{-1}(a/b)$$

$$E_{Net} = \frac{9 \times 10^9 \, Q}{b^2(a^2+b^2)} \left[4b^2 \cos\left[\tan^{-1}(a/b)\right] - (a^2+b^2) \right]$$

x — x —— x —— x —— x — x