IQRA NATIONAL UNIVERSITY

Department of Electrical Engineering

Digital Signal Processing

Name: Kaleem Ullah

ID: 13170

Submitted to: Engr Pir Meher Ali Shah Sir

Semester: 8th

Digital Signal Processing

Kaleem
13170

Question No # 01

=> Consider the following analog signal

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

=> Part (a)

(i) Determine the minimum sampling rate required to avoid aliasing.

Minimum sampling rate

$x_a(t) \; 3\cos 100\pi t + 4\sin 200\pi t$

$F_1 = 50\,Hz$

$F_2 = 100\,Hz$

$F_3 = 100\,Hz$

F_s is maximum than F_1 $F_1 = 50$

$F_1 = 50$ is minimum sampling rate to aviod aliasing

$f_b = 2\,F_{max}$

F_2 is Max, $F_2 = 100\,Hz$

13170

$\Rightarrow$ F_2 is max , $F_2 = 100\,Hz$

$F_b = 2 \times 100$

$F_b = 200$

(ii) we have

$F_b = 100\ Hz$

So $f_1' = \frac{f_1}{F_S} = \frac{50}{100} = 0.5\,Hz$

F_2 becomes

$f_2' = \frac{F_2}{F_b} = \frac{100}{100} = 1\ Hz$

So $w_1' = 2\pi f_1'$

$w_1' = 2\pi \times 0.5$

$w_1' = \pi$

and

$w_2' = 2\pi f_2'$

$w_2' = 2\pi \times 1$

$w_2' = 2\pi$

⇒ 13170

⇒ $x[n] = 3\cos 100\pi t + 4\sin 200\pi t$

$x[n] = 3\cos \pi t + 4\sin 2\pi t$

(iii) we can construct the orignal signal and also frequency component since at 50 Hz and 100 Hz are present in the sampled signal

The signal we can recover is

$y_a(t) = 3\cos \pi t + 4\sin 2\pi t$

$x_a(t)$ orignal signal is diffrent from it due to low sampling rate used distortion of the orignal analog signal was caused by the aliasing effect.

$\Rightarrow$ 13170

Question 1 Part (B)

$\Rightarrow$ Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$\Rightarrow$ This is signal is sampled at the rate

$F_s = 2Hz$

Solution:- As we know

$$F_s = \frac{1}{T}$$

$$\Rightarrow T = \frac{1}{F_s}$$

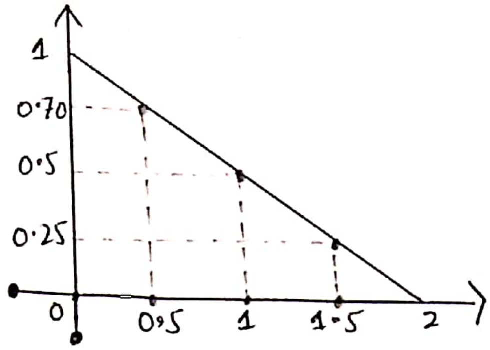
$$T = \frac{1}{2}$$

$$T = 0.5 \text{ sec}$$

(i) Draw the sampled signal.

$x n$	0.5^0
0	1
0.5	0.707
1	0.5
1.5	0.25

⟹ 13170



(ii) The samples of the signals are intended to carry 3 bits per samples Determine the quantazation level and qustazation resolution to qustized sampled signal achieved in part (1)

Solution: ⟹ n=3 bits / sample

First we have to find quantization level:

$$L = 2^n$$

$$L = 2^3$$

$$L = 8$$

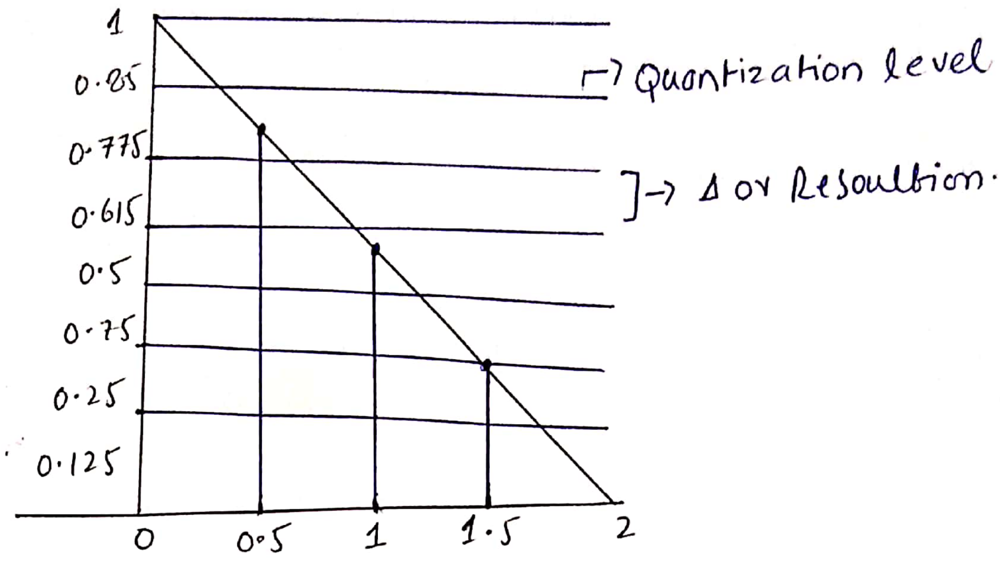
Quantization resolution $(\Delta) = \frac{x_{max} - x_{min}}{L}$

$$\Delta = \frac{1-0}{8}$$

$$\Delta = \frac{1}{8} \Rightarrow \Delta = 0.142$$

⇒ 13170

⇒



(iii) perform the process of truncation rounding off number of the sampled signal and find the quantization error for each of the sampled data Express your answer in tabuler form.

n	Discrete time signal	Rounding	Truncation	$e_q[n] = x_E[n] - x[x]$
0	1	1	1	0
1	0.87	0.8	0.8	-0.0
2	0.75	0.7	0.7	0.0
3	0.65	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.37	0.3	0.4	-0.1
6	0.25	0.2	0.3	-0.1
7	0.12	0.1	0.1	0.0

⇒ 13170

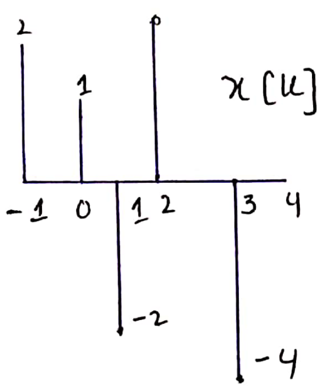
Question # 02

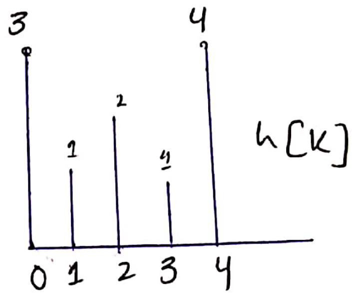
⇒ Part (a) Determine the response of the System to the following input with given impulse response

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3 -4\}$$

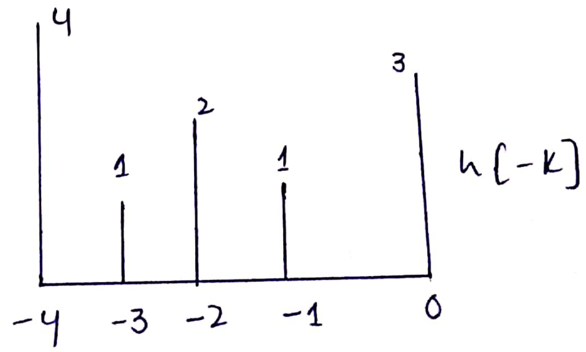
$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution:-





⇒ Convert h[k] into h[-k]:



⇒ As $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$.

→ 13170

⇒ For $n = 0$

$$y[n] = \sum_{k=-\infty}^{0} x[k]\, h[n-k]$$

$$y[0] = \sum_{k=-1}^{0} x[k]\, h[-k]$$

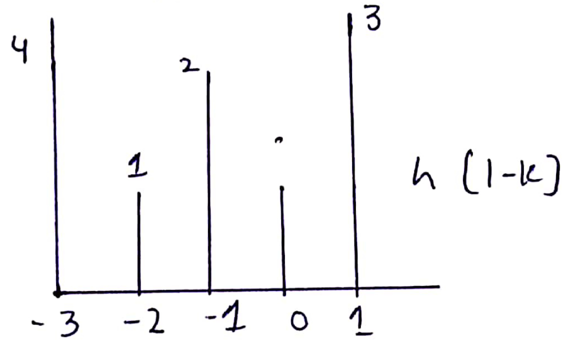
$$= x[-1]\, h(-1) + x(0)\, h(0)$$

$$= (2)(1) + (1)(3)$$

$$= 2 + 3$$

$$y[0] = 5$$

⇒ For $n = 1$



$$y[1] = \sum_{k=-1}^{1} x[k]\, h[1-k]$$

$$= x(-1)\, h(-1) + x(0)\, h(0) + x(1)\, h(1)$$

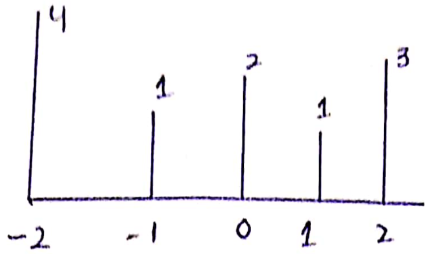
$$= (2)(2) + (1)(1) + (-2)(3)$$

$$= 4 + 1 - 6 \quad \Rightarrow \boxed{y(1) = -1}$$

$$= 5 - 6$$

⇒ 13170

⇒ For $n = 2$



$n(2-k]$

$$\Rightarrow \; y[2] = \sum_{k=1}^{2} x[k]\,h[2-k]$$

$= x(1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2)$

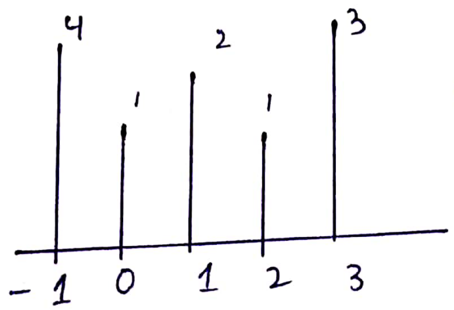
$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$

$= 2+2-2+9$

$= 2+9$

$y[2] = 11$

⇒ For $n = 3$:



$h[3-k)$

$$\Rightarrow \; y(3) = \sum_{x=-1}^{3} x[n]\,h[3-k]$$

$= x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2)$
$+ x(3)\,h(3)$

Scanned with CamScanner

$\Rightarrow y[3] = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 8 + 1 - 4 - 12 + 3$

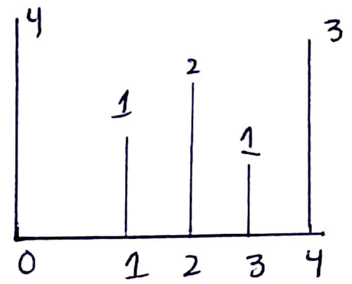
$= 1 - 4 - 4 + 3$

$= 1 - 8 + 3$

$y[3] = -7 + 3$

$= -4$

$\Rightarrow n = 4$



$\Rightarrow y[4] = \sum_{k=0}^{3} x[k]\, h[4-k]$

$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$

$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$

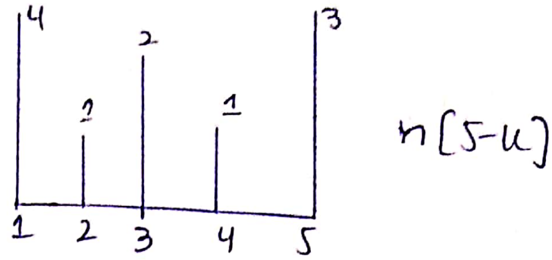
$= 4 - 2 + 6 - 4$

$= -2 + 6$

$y[4] = 4$

$\Rightarrow$ 13170

$\Rightarrow$ $n = 5$



$$y[5] = \sum_{k=1}^{3} [k]\, h[5-k]$$

$$= x(1)\,h(1) + x(2)\,h(2) + x(3)\,h(3)$$

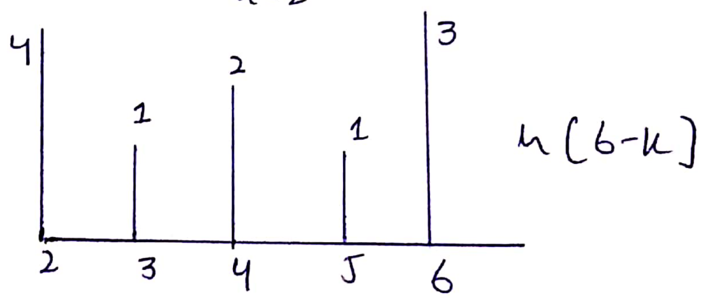
$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$= -8 + 3 - 8$$

$$= -13$$

$\Rightarrow$ $n = 6$

$$y[6] = \sum_{k=2}^{3} x[k]\, h[6-k]$$



$$y[6] = x(2)\,h(2) + x(3)\,h(3)$$

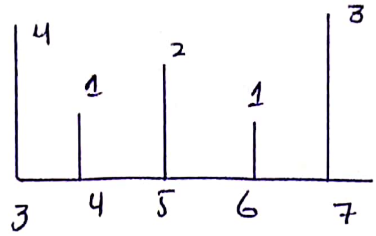
$$= (3)(4) + (-4)(1)$$

$$= 12 - 4$$

$$= 8$$

$\Rightarrow$ 13170

$\Rightarrow \quad n = 7$



$h[7-k]$

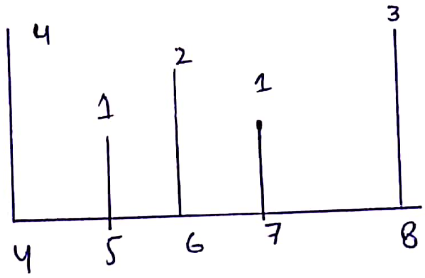
$$y(7) = \sum_{k=3}^{3} x[k]\, h[7-k]$$

$$= x(3)\, h(3)$$

$$= (-4)\ (4)$$

$$y(7) = -16$$

$\Rightarrow \quad n = 8$



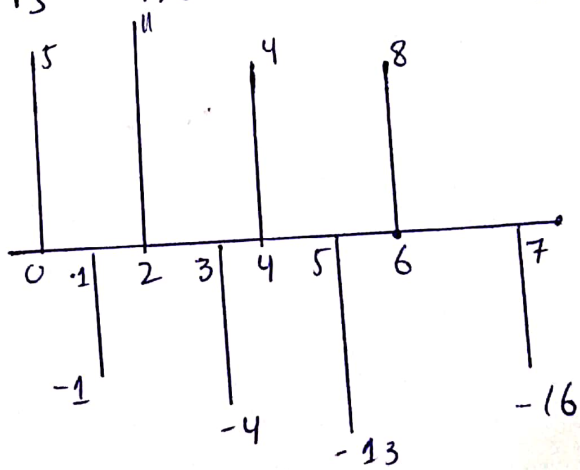
$h[8-k]$

$$y[8] = 0$$

There is no over lapping in $n = 8$

So

$y(n) =$



=> 13170

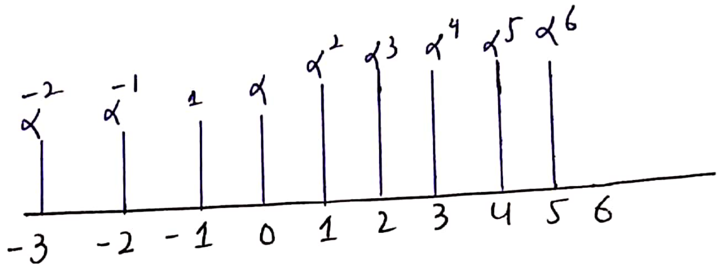
Question No # (2)

Part (b) Compute the Convolution $y(n)$ of the following Signal

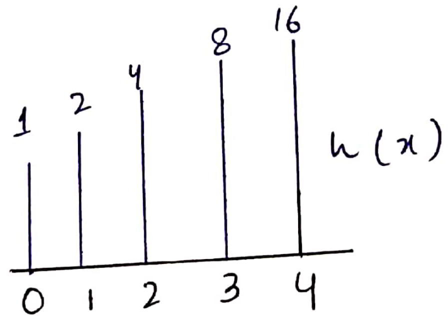
$$x(n) \begin{cases} \alpha^{n+1} & , -3 \le n \le 5 \\ 0 & , \text{else} \end{cases}$$

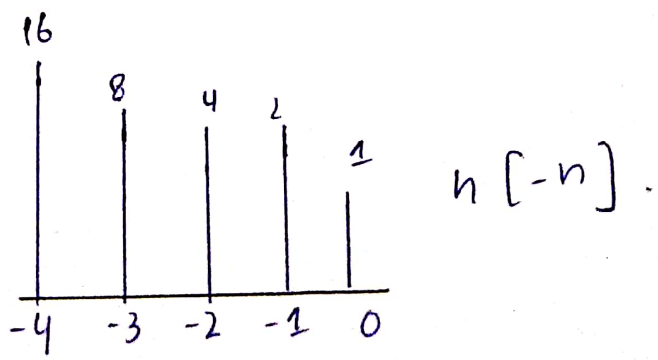
$$h(n) = \begin{cases} 2^n & , 0 \le n \le 4 \\ 0 & \text{else} \end{cases}$$

Solution :- $x(n) = \{ \alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6 \}$



$h(x) = \{ 1, 2, 4, 8, 16 \}$





$\Rightarrow$ 13170

$\Rightarrow \quad y[-3] = (\alpha^{-2})(1)$

$y[-3] = \alpha^{-2}$

$y[-2] = (\alpha^{-1})(1) + (\alpha^{-2})(2)$

$y[-2] = 2^{-1} + \alpha^{-2}$

$y[-2] = 2^{-3}$

$y[-1] = (\alpha^{1})(1) + (2)(4) + (2^{-1})(4) + (2^{-2})(8)$

$y[-1] = (1 \times 10) + 2\alpha^{-1} + 4\alpha^{-2}$

$y[-1] = 16\alpha^{-3} + 1$

$y[-1] = 16\alpha^{-3} + 1$

$y[0] = (8)(\alpha^{-2}) + (\alpha^{-1})(4) + (1)(2) + (\alpha)(1)$

$y[0] = 8\alpha^{-2} + 4\alpha^{-1} + 2 + \alpha$

$y[0] = 12\alpha^{-3} + 2 + \alpha.$

$\Rightarrow$ 13170

$\Rightarrow$ $y[1] = (\alpha^{-1})(8) + (1)(4) + (\alpha)(2) + (\alpha^2)(1)$

$= 8\alpha^{-1} + 4 + 2\alpha + \alpha^2$

$y(2) = (\alpha^{-1})(16) + (1)(8) + (\alpha)(4) + (\alpha^2)(2)$
$+ (\alpha^3)(1)$

$y(2) = 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$

$y[3] = (1)(16) + (\alpha)(8) + (\alpha^2)(4) + (\alpha^3)(2) + (\alpha^4)(1)$

$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$

$y(4] = (\alpha)(16) + (\alpha^2)(8) + (\alpha^3)(4) + (\alpha^4)(2) +$
$(\alpha^5)(1)$

$y[4] = 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$

$y[5] = (\alpha^2)(16) + (\alpha^3)(8) + (\alpha^4)(4) + (\alpha^5)(2) +$
$(\alpha^6)(1)$

$y(5) = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$

$y[6] = (\alpha^3)(16) + (\alpha^4)(8) + (\alpha^5)(4) +$
$(\alpha^6)(2) +$

$y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

$\Rightarrow$ 13170

$\Rightarrow$ $y[7] = (16)(\alpha^4) + (\alpha^5)(8) + (\alpha^6)(4)$

$y[7] = 16\alpha^4 + 8\alpha^5 + 4\alpha^6$

$y[8] = (\alpha^5)(16) + (\alpha^6)(8)$

$y[8] = 16\alpha^5 + 8\alpha^6$

$y[9] = 16\alpha^6$

$y[10] = 0$

$\Rightarrow$ There is no overlap in $y[10]$

Question No (3)

Determine the z transform of the following signals and also sketch its region of convergence.

(i) $x(n) = \begin{cases} (\frac{1}{4})^n, & n \geq 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$

Solution: The z transform is

$x(n) = a^n u(n)$

⇒ 13.1.70

$\Rightarrow \quad x(z) = \frac{1}{1-\alpha z^{-1}} \quad ROC |z|$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

using geometric Series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - (1-\frac{1}{3}z - \frac{1}{4}z^{1} + \frac{1}{12}z^{-1}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - 1 + \frac{1}{3}z + \frac{1}{4}z^{1} + \frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1 + 1/12}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\frac{13}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)} \text{, so } \boxed{ROC \ \frac{1}{4} < |z| < 3}$$

$\Rightarrow$ 13170

Question # 03 Part (B)

$$\Rightarrow \quad x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{else} \end{cases}$$

Solution: So the z-transform is

$$x(n) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$\frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$\frac{1 - 3z^{-1} - 1 - \frac{1}{2} t^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

$$\frac{-\frac{5z^{-1}}{2}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

$\Rightarrow$ Hence the ROC is $|z| > 3$

$\therefore -3z^{-1} - \frac{1}{2} z^{-1}$

$\therefore \frac{-6z^{-1} - z^{-1}}{2}$

$\therefore \frac{-5z^{-1}}{2}$

The End

$\longleftrightarrow$