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Subject :- Applied Calculus

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MID TERM EXAM.

Q1) The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \end{cases}$$

$$t^2 \quad 0 \leq t \leq 3$$

$$2t + 3 \quad 3 < t \leq 4$$

$$12 \quad t > 4$$

- a) State any point of discontinuity
 b) Find, if they exist
 i) $\lim_{t \rightarrow 3} g$

Sol:-

- a) To check possibility of the discontinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L

$$\lim_{h \rightarrow 0} g(1+h) = \lim_{h \rightarrow 0} (1+h)^2$$

$$= \lim_{h \rightarrow 0} 1 + h^2 + 2h$$

Apply limits

$$= 1 + 0^2 + 2(0)$$

$$= 1$$

For L.H.L

$$\lim_{h \rightarrow 0} g(1-h) = 2t+3$$

$$= \lim_{h \rightarrow 0} 2(1-h) + 3.$$

$$= \lim_{h \rightarrow 0} 2 - 2h + 3$$

Apply limit

$$= 2 - 2(0) + 3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

→ Now at $t = 4$

$$g(4) = 2(4) + 3$$

$$= 8 + 3$$

$$= \underline{11}$$

For R.H.L

$$\lim_{h \rightarrow 0} g(1+h) = \lim_{h \rightarrow 0} 2(1+h) + 3$$

$$= \lim_{h \rightarrow 0} 2 + 2h + 3$$

Apply limits

$$= 2 + 2(0) + 3 \Rightarrow 5$$

For L.H.L

$$\lim_{h \rightarrow 0} g(1-h) = 12$$

$g(4) = \text{R.H.L} \neq \text{L.H.L}$
point of discontinuity is at $t=4$

5) Find, if they exist

i) $\lim_{t \rightarrow 3} g$

For $g(t) = t^2$

$$\text{R.H.L} \quad \lim_{h \rightarrow 3} g(1+h) = \lim_{h \rightarrow 3} (1+h)^2$$

$$= \lim_{h \rightarrow 3} 1 + h^2 + 2h$$

Apply limits

$$= 1 + 3^2 + 2(3) \Rightarrow 16$$

L.H.L

$$\begin{aligned}\lim_{h \rightarrow 3} g(1-h) &= \lim_{h \rightarrow 3} 2t + 3 \\ &= \lim_{h \rightarrow 3} 2(1-h) + 3 \\ &= \lim_{h \rightarrow 3} 2 - 2h + 3\end{aligned}$$

Apply limit

$$\begin{aligned}&= 2 - 2(3) + 3 \\ &= 2 - 6 + 3 \\ &= -1\end{aligned}$$

R.H.L \neq L.H.L (do not exist since L.H.L is -ve)

(Q2) Find the Maclaurin's Series for
 $y(x) = x^2 + \sin x$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

Let $y(x) = x^2 + \sin x$

$$y'(x) = 2x + \cos x$$

$$y''(x) = 2 - \sin x$$

$$y'''(x) = -\cos x$$

Put $x=0$ in all
function

$$y(0) = 0$$

$$y'(0) = 1$$

$$y''(0) = 2$$

$$y'''(0) = -1$$

Putting value in formula

$$= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1)$$

$$= 0 + x + x^2 - \frac{x^3}{6}$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} \quad \text{Ans -}$$

Q3) i) Find y'' given

$$1 + xy = x^2 + y^2$$

Sol: $1 + xy = x^2 + y^2$ — (1)

diff eq (1) b/s w.r.t "x"

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$= 0 + \left(x \cdot \frac{d}{dx}y + y \cdot \frac{d}{dx}x\right) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot \frac{d}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot y' + y = 2x + 2y \cdot y' \quad \text{H#1 (17/16)}$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$= x \cdot y' - 2y \cdot y' = 2x - y$$

$$= y'(x - 2y) = (2x - y)$$

$$= y' = \frac{2x - y}{x - 2y}$$



Differ again w.r.t "x"

$$y'' = \frac{d}{dx} \left(\frac{2x-y}{x-2y} \right)$$

$$= \frac{(x-2y) \frac{d}{dx}(2x-y) - (2x-y) \frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2) \left(-\frac{dy}{dx} \right) - (2x-y) \left(1 - 2 \frac{dy}{dx} \right)}{(x-2y)^2}$$

$$= \frac{(2x-4y)(-y') - (2x-4x \frac{dy}{dx} - y + 2yy')}{(x-2y)^2}$$

$$y'' (x-2y)^2 = 2xy' + 2yy' - 2x + y$$

$$y'' (x-2y)^2 = \left(\frac{2x-y}{x-2y} \right) (2x-2y) - 2x + y$$

$$y'' = \frac{\left(\frac{2x-y}{x-2y} \right) (2x+2y) - 2x + y}{(x-2y)^2}$$

Ans

ii) Find y' by using logarithmic differentiation.

$$y = x^3(1+x)^9 e^{6x}$$

Sol:- Taking \ln on b/s

$$\ln y = \ln (x^3(1+x)^9 \cdot e^{6x})$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + 6x \ln e$$

Diff w.r.t x

$$\frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln x + \frac{d}{dx} 9 \ln (1+x) + \frac{d}{dx} 6x \ln e$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}} \cdot 6$$

$$\frac{dy}{dx} = y \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 e^{6x} \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

Ans.